ELECTRIC CIRCUITS

THEORY AND APPLICATIONS

VOLUME I

SHORT-CIRCUIT CALCULATIONS

AND

STEADY-STATE THEORY

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STEADY-STATE THEORY

BY

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PREFACE

The purpose of this treatise is to present the engineering aspects of circuit theory. Although theoretical, the language and viewpoint of the book are those of the engineer. It gives the methods and tools necessary for the analysis of modern power-circuit problems.

The book has grown out of the author's experience as a teacher and an electrical engineer. It is primarily intended as a textbook for the course "Electric Circuits" included in the postgraduate curriculum of the Electrical Engineering Department of the Massachusetts Institute of Technology. There has been a long-felt need for such a text. It is hoped, however, that the book also may prove of use to students of similar courses at other institutions, to electrical-engineering students in general, and to many practising electrical engineers.

Numerous illustrative examples are worked out in the text. These examples are, wherever practicable, based on actual engineering data and are representative of the type of problems with which the electrical engineer to-day deals.

The author wishes to express his appreciation and thanks to Mr. G. H. Arapakis, Instructor in Electrical Engineering at the Massachusetts Institute of Technology, who read the manuscript, offered many valuable suggestions, and who worked out and checked several of the numerical problems. Thanks are also due to Dr. E. A. Guillemin, Instructor in Electrical Engineering, for reading and criticizing parts of the manuscript, to Messrs. E. Bramhall, L. A. Bingham, C. V. Bullen, O. W. Walter, R. B. Wright, and D. S. Young, formerly graduate students of electrical engineering, for calculating the numerical data used in the example in the chapter on synchronous-machine charts, and to Mr. H. F. Goodwin of Jackson and Moreland, Engineers, Boston, Mass., for preparing some of the more important drawings. Last, but not least, the writer is indebted to the authors of those technical papers and manufacturers' publications which have served as sources of material during the preparation of the manuscript.

O. G. C. Dahl.

Cambridge, Mass.,
July, 1928.

CONTENTS

SHORT-CIRCUIT CALCULATIONS
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CHAPTER I

CALCULATION OF SHORT-CIRCUIT CURRENTS IN NETWORKS

There are, in general, two problems in network solution with which the electrical engineer has to deal, namely, (1) the determination of voltages, currents, and power for normal operating conditions and (2) the determination of voltages and currents during short circuits.

Analytical solutions of complicated alternating-current networks may be extremely laborious. This is particularly so when a network is fed by several generating stations and supplies numerous loads. Three methods of attack are available: (*a*) solution by simultaneous equations, (*b*) solution by trial and error, (*c*) solution by simplification of the network. In the following, the first two methods will be only briefly indicated. For a more thorough treatment, the reader is referred to other texts[1] and to the numerous papers[2] on the subject in the technical press. The method of solution by simplification will be discussed

[1] Herzog, J. and C. Feldmann, "Die Berechnung Elektrischer Leitungsnetze in Theorie und Praxis," Julius Springer, Berlin, 1921.

Woodruff, L. F. "Principles of Electric Power Transmission and Distribution, Chap. XIV, John Wiley and Sons, Inc., New York, 1925.

[2] See, for instance:

Woodward, W. R., R. D. Evans, and C. L. Fortescue, "Calculating Short-circuit Currents in Networks," *Elec. Jour.*, p. 344, 1919. This article is divided into three parts as follows:

1. "Testing with Miniature Networks," by Woodward.
2. "Analytical Solutions," by Evans.
3. "Development of Analytical Solutions," by Fortescue.

The methods presented in the last two parts are applicable also under normal operating conditions.

in more detail, as it is almost universally applied in practical short-circuit calculations.

The first method involves the setting up of a system of vector equations by applying Ohm's and Kirchhoff's laws. Upon having established the equations, the unknowns are determined by simultaneous solutions. If the number of equations is large, this process is an extremely cumbersome one, although the process of elimination may be shortened by the use of determinants.

In order to use the second method by which the correct solution is approached by steps, it is necessary to assume values of current or power in one or more branches of the network. It is impossible definitely to outline the procedure to be followed without reference to some specific problem. The calculations, however, would as a rule, be in conformance with the following general scheme:

Making use of the assumed values, calculations are carried out between points at which certain electrical conditions are definitely known. This sometimes (particularly in simple loop circuits) involves coming back to the starting point. If the electrical quantities calculated at the points at which conditions are actually known check with the latter, a correct solution is obtained. This shows, then, that the original assumptions in regard to values of current or power (or both) are correct. If, on the other hand, the known electrical conditions are not checked, the original assumptions are obviously in error. The discrepancies between calculated and known conditions may this time be used as a guide in correcting the initially assumed values of current or power, and the calculations repeated. If, upon recomputation, discrepancies between calculated quantities and known quantities still are present, a second adjustment of the assumed values with a subsequent recomputation is necessary.

Thomälen, A., "Zur zeichnerischen Behandlung beliebiger Leitungsnetze," *Elektrotek. Z.*, p. 694, 1921.

Chapman, F. T., "The Calculation of Direct-current and Alternating-current Networks," *Elec. Rev.* (London), p. 486, 1923.

Blake, D. K., "Alternating-current Secondary Networks," *General Elec. Rev.*, p. 391, 1923.

Evans, R. D., "The Analytical Solution of Networks," *Elec. Jour.*, pp. 149 and 207, 1924.

Richter, H., "Evolution of Alternating-current Secondary Networks," *Elec. Jour.*, p. 320, 1925.

Chrustschoff, W., "Beitrag zur Berechnung elektrischer Leitungsnetze." *Elektrotek. Z.*, p. 1405, 1927.

By proceeding in this manner, a correct solution is eventually reached. With some practice in the trial-and-error method, solutions sufficiently accurate for engineering purposes will, as a rule, be obtained after one or two recalculations.

The two methods just described are general and applicable to any type of network, independent of whether the network is fed by one or more generating stations and supplies one or more loads. The third method, involving simplification of the network itself, however, is somewhat limited in its application. It is generally applicable only when the network is fed by a single generating station and supplies a single load. The solution by this method is obtained by reducing the network between the generating station and the load to a single equivalent impedance. This can always be done even if the network is complicated.

So far, the discussion has been devoted to the solution of networks for normal operating conditions. When it is desired to determine voltages and currents during short circuits, the same general methods may be applied.[1] Usually, however, the solution for short-circuit conditions is a good deal simpler than for normal conditions.

The determination of short-circuit currents is a frequently recurring problem in electrical engineering. It is necessary to know the short-circuit currents at various points in the network so that the proper size of circuit breakers may be selected. During times of short circuit, the circuit breakers involved are called upon to stand the maximum short-circuit current which will flow and to interrupt the current after the elapse of a certain time. The currents which the breakers have to interrupt are usually a good deal smaller than the short-circuit currents which they carry initially. Even so, however, these currents may be extremely high and easily equal to many times the current which the breaker normally handles. The operation of many of the protective relays used in networks today also depends upon short-circuit currents. Hence, in order to enable the engineer to select and

[1] An excellent treatise on the general matter of short-circuit currents in networks is "Überströme in Hochspannungsanlagen," by J. Biermanns, Julius Springer, Berlin, 1926.

The following papers discuss special short-circuit problems:

Bekku, S., "Calculation of Short-circuit Ground Currents on Three-phase Power Networks," *Gen. Elec. Rev.*, p. 472, 1925.

Lewis, W. W., "Single-phase Short-circuit Calculations," *Gen. Elec. Rev.*, p. 479, 1925.

install the proper relays and to determine their setting, knowledge of the values of the short-circuit currents is necessary.

Since, as already mentioned, short-circuit calculations have to be performed frequently, the methods should be as direct and simple as possible. It is not essential to obtain the short-circuit currents with extreme accuracy. If it were attempted to obtain entirely rigorous solutions, the methods would become exceedingly complicated and require an undue amount of time, to say the least. The fact really is that it would be impossible in most cases to obtain rigorous solutions. Experience has shown that the approximate methods which are in common use are sufficiently accurate for engineering purposes. These methods have become standardized. They are simple and readily applicable even to complicated networks.

When a short circuit occurs, the network will, as a rule, be supplying power to several loads. The first assumption which is made, in order to facilitate the short-circuit calculations, is that the load currents may be ignored in comparison with the short-circuit currents. As a rule, the short circuit is confined to a single point only at a time, and this is always assumed in the calculations. It is further assumed that the impedance of the short circuit itself is zero, *i.e.*, a short circuit is always considered to be a "dead short circuit" in the true sense of the word.

If the network on which the short circuit occurs were supplied by one generating station only, it is obvious that the method of simplification would be immediately applicable. In order to extend the applicability of this method also to the general case where several stations supply the network, it is assumed that the induced voltages at the various generating stations are equal both in magnitude and phase. Of course, this will not be quite true, but the error introduced by this assumption should not be a serious one. When the assumption is made, however, that the generator voltages are equal, all generators supplying the system may be assumed connected to a hypothetical bus at which the common voltage is maintained. The network may then be reduced to a single equivalent impedance between the hypothetical bus and the point of short circuit, and the solution hence obtained by the method of simplification.

In reducing the network, the impedance of lines, transformers, and generators should all be included. It has become customary to neglect resistance, leakance, and capacitance of lines and

feeders and to consider their reactance only. Similarly, the resistance of transformers and generators is neglected. The transformers are represented by their equivalent reactance, their exciting currents being ignored. The reactance assigned to the generators will be discussed in more detail below. Since the entire network contains reactances only, the handling of complex quantities is avoided during the process of simplification, a fact which evidently reduces the amount of labor in no small degree.

Simplification of Networks.—There are three transformations available which may be used during the process of reducing a network to a single equivalent impedance between two points. Usually it is necessary to simplify the network by steps through a repeated application of transformations. The three methods of transformation are given below.

Δ-Y *Transformation.*—Whenever three impedances form a Δ or a three-cornered mesh, it is possible to convert this circuit into a Y or a three-cornered star.[1] The two circuits (Fig. 1) will be equivalent as far as conditions at the terminals are concerned.

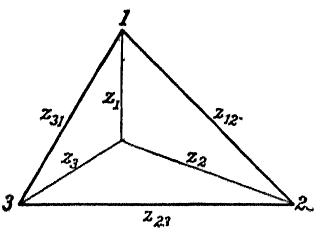


FIG. 1.—Equivalent Y- and Δ-impedances.

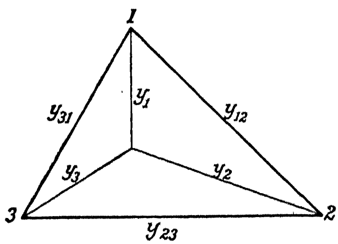


FIG. 2.—Equivalent Y- and Δ-admittances.

This means that the two circuits will offer identical impedances between any pair of terminals and will, for the same applied voltage, absorb the same amount of active and reactive power.

In terms of impedances, the conversion formulas are as follows:

$$Z_1 = \frac{Z_{31}Z_{12}}{Z_{12} + Z_{23} + Z_{31}} = \frac{Z_{31}Z_{12}}{\Sigma Z} \qquad (1)$$

$$Z_2 = \frac{Z_{12}Z_{23}}{Z_{12} + Z_{23} + Z_{31}} = \frac{Z_{12}Z_{23}}{\Sigma Z} \qquad (2)$$

$$Z_3 = \frac{Z_{23}Z_{31}}{Z_{12} + Z_{23} + Z_{31}} = \frac{Z_{23}Z_{31}}{\Sigma Z} \qquad (3)$$

[1] KENNELLY, A. E., "The Equivalence of Triangles and Three-pointed Stars in Conducting Networks," *Elec. World and Eng.*, Vol. XXXIV, p. 413, 1899.

In terms of admittances (see Fig. 2), the conversion formulas become

$$y_1 = \frac{y_{12}y_{23} + y_{23}y_{31} + y_{31}y_{12}}{y_{23}} = \frac{N}{y_{23}} \tag{4}$$

$$y_2 = \frac{y_{12}y_{23} + y_{23}y_{31} + y_{31}y_{12}}{y_{31}} = \frac{N}{y_{31}} \tag{5}$$

$$y_3 = \frac{y_{12}y_{23} + y_{23}y_{31} + y_{31}y_{12}}{y_{12}} = \frac{N}{y_{12}} \tag{6}$$

Y-Δ Transformation.—If the given circuit is a Y or three-cornered star, it may be changed into an equivalent Δ or three-cornered mesh. In terms of impedances (refer to Fig. 1), the conversion formulas are

$$Z_{12} = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3} = \frac{N}{Z_3} \tag{7}$$

$$Z_{23} = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1} = \frac{N}{Z_1} \tag{8}$$

$$Z_{31} = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_2} = \frac{N}{Z_2} \tag{9}$$

The conversion formulas in terms of admittances become

$$y_{12} = \frac{y_1y_2}{y_1 + y_2 + y_3} = \frac{y_1y_2}{\Sigma y} \tag{10}$$

$$y_{23} = \frac{y_2y_3}{y_1 + y_2 + y_3} = \frac{y_2y_3}{\Sigma y} \tag{11}$$

$$y_{31} = \frac{y_3y_1}{y_1 + y_2 + y_3} = \frac{y_3y_1}{\Sigma y} \tag{12}$$

Star-mesh Transformation.—Any star circuit may be converted into its equivalent mesh circuit independent of the number of rays in the originally given star. The converse theorem does not hold, *i.e.*, it is not, in general, possible to convert a general mesh circuit into its equivalent star.[1]

Figure 3 shows a star with n rays and its equivalent mesh circuit. The number of impedances in the mesh is $\frac{n}{2}(n - 1)$. In

[1] See the paper "A New Network Theorem," by A. Rosen, *Jour. I.E.E.* (London), Vol. 62, p. 916. In this paper, Mr. Rosen proves the general conversion from a star circuit to a mesh circuit. He derives the necessary formulas and also shows that the mesh, in general, cannot be converted to a star.

terms of impedances, the general conversion formulas from star to mesh are

$$Z_{12} = Z_1Z_2\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}\right) = Z_1Z_2\Sigma\frac{1}{Z} \quad (13)$$

$$Z_{13} = Z_1Z_3\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}\right) = Z_1Z_3\Sigma\frac{1}{Z} \quad (14)$$

$$Z_{mn} = Z_mZ_n\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}\right) = Z_mZ_n\Sigma\frac{1}{Z} \quad (15)$$

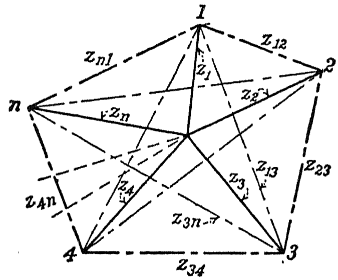


FIG. 3.—Equivalent mesh impedances corresponding to a given star circuit.

In terms of admittances, the corresponding equations become

$$y_{12} = \frac{y_1y_2}{y_1 + y_2 + \cdots + y_n} = \frac{y_1y_2}{\Sigma y} \quad (16)$$

$$y_{13} = \frac{y_1y_3}{y_1 + y_2 + \cdots + y_n} = \frac{y_1y_3}{\Sigma y} \quad (17)$$

$$y_{mn} = \frac{y_my_n}{y_1 + y_2 + \cdots + y_n} = \frac{y_my_n}{\Sigma y} \quad (18)$$

Since these equations are general, they will obviously also hold for the Y-Δ transformation. It is easily shown that, if applied to this case, the general impedance equations reduce to equations (7) to (9), inclusive, and the general admittance equations to equations (10) to (12), inclusive.

EXAMPLE 1

This example illustrates the solution of a simple network problem by the method of simplification.

Statement of Problem

Figure 4 shows a single-wire diagram (simplified) of a railway electrification with trolleys, feeders, and substations.

The assumptions, much simplified, are as follows:

All circuits single phase.

Distribution at 44,000 volts, approximately; trolley at 11,000 volts, approximately; 25 cycles.

Transformers at each substation rated at 8,000 kv.-a., 4 per cent resistance, 7 per cent reactance. Neglect excitation.

Trains, maximum input 6,000 kv.-a. at 80 per cent power factor when trolley voltage is 11,000 volts.

All 44,000-volt transmission lines two No. 00 copper conductors spaced 6 ft.

Trolley No. 000 copper conductor. Rail return. Neglect resistance of rail and ground return. In computing trolley inductance, consider return to be equal diameter conductor spaced 40 ft. Note that substation 3 is fed by a single transmission feeder, while substations 1 and 2 are fed by two identical feeders. These feeders may be considered to be on separate pole lines.

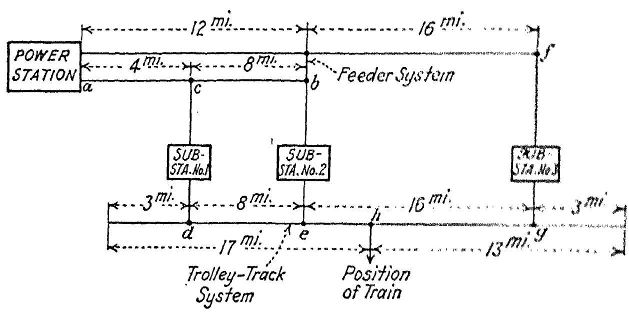


FIG. 4.—Simplified layout of a single-phase railway electrification.

Consider train in the position shown and with controller set so that impedances are those corresponding to maximum input. Compute voltage, current, and power at the train when the bus voltage at the power station is strictly 44,000 volts.

Solution

Circuit Constants:

High-tension Feeders (No. 00 copper):

Resistance[1] = 0.822 ohm/loop-mile

Reactance[1] = 0.630 ohm/loop-mile

Trolley-track System:

Resistance[1] = 0.326 ohm/loop-mile

$$\text{Reactance} = 2\pi f\left(741 \log_{10} \frac{D}{r} + 80.5\right)10^{-6}$$

$$= 2\pi \times 25\left(741 \log_{10} \frac{40 \times 12}{0.2048} + 80.5\right)10^{-6}$$

$$= 0.404 \text{ ohm/loop-mile}$$

[1] Values taken from tables in "Handbook for Electrical Engineers," by H. PENDER, John Wiley and Sons, Inc., New York, 1917.

Substation Transformers:

$$I_{\text{rated}} = \frac{8{,}000 \times 1{,}000}{44{,}000} = 181.8 \text{ amp.}$$

$$\left.\begin{aligned}\text{Resistance} &= \frac{44{,}000 \times 0.04}{181.8} = 9.68 \text{ ohms}\\ \text{Reactance} &= \frac{44{,}000 \times 0.07}{181.8} = 16.92 \text{ ohms}\end{aligned}\right\} \text{Referred to high-tension side}$$

Train:

$$I_{\text{rated}} = \frac{6{,}000 \times 1{,}000}{11{,}000} = 546 \text{ amp.}$$

$$\text{Impedance} = \frac{11{,}000}{546}\underline{/\cos^{-1} 0.8} = 20.14\underline{/36^\circ.9} \text{ ohms}$$

$$\text{Resistance} = 20.14 \times 0.8 = 16.11 \text{ ohms}$$

$$\text{Reactance} = 20.14 \times 0.6 = 12.09 \text{ ohms}$$

Simplification of the Network.—In reducing the system to a single, equivalent impedance between the high-tension bus of the generating station and the train, all impedances will be referred to the high-tension side. Referring to Fig. 4 and to Fig. 5*a*, the impedances of the various sections become

$$\begin{aligned}
Z_{ab} &= (0.822 + j0.630)12 = 9.864 + j7.560 \text{ ohms}\\
Z_{ac} &= (0.822 + j0.630)4 = 3.288 + j2.520 \text{ ohms}\\
Z_{bc} &= (0.822 + j0.630)8 = 6.576 + j5.040 \text{ ohms}\\
Z_{bf} &= (0.822 + j0.630)16 = 13.152 + j10.080 \text{ ohms}\\
Z_{cd} &= Z_{be} = Z_{fg} = 9.68 + j16.992 \text{ ohms}\\
Z_{de} &= (0.326 + j0.404)4^2 \times 8 = 41.76 + j51.68 \text{ ohms}\\
Z_{eh} &= (0.326 + j0.404)4^2 \times 6 = 31.32 + j38.76 \text{ ohms}\\
Z_{gh} &= (0.326 + j0.404)4^2 \times 10 = 52.20 + j64.60 \text{ ohms}
\end{aligned}$$

By adding series impedances wherever possible, the network in Fig. 5*b* is obtained. The two △'s *abc* and *beh* will now be converted to Y's, as indicated by the dotted lines.

Conversion of *abc*

$$\begin{aligned}
Z_{ab} &= 9.864 + j7.560 = 12.44\underline{/37^\circ.5}\\
Z_{ac} &= 3.288 + j2.520 = 4.14\underline{/37^\circ.5}\\
Z_{bc} &= 6.576 + j5.040 = 8.30\underline{/37^\circ.5}\\
\hline
\Sigma Z &= 19.728 + j15.120 = 24.88\underline{/37^\circ.5}
\end{aligned}$$

$$Z_{ai} = \frac{4.14\underline{/37^\circ.5} \times 12.44\underline{/37^\circ.5}}{24.88\underline{/37^\circ.5}} = 2.073\underline{/37^\circ.5} = 1.644 + j1.261$$

$$Z_{bi} = \frac{12.44\underline{/37^\circ.5} \times 8.30\underline{/37^\circ.5}}{24.88\underline{/37^\circ.5}} = 4.15\underline{/37^\circ.5} = 3.288 + j2.522$$

$$Z_{ci} = \frac{4.14\underline{/37^\circ.5} \times 8.30\underline{/37^\circ.5}}{24.88\underline{/37^\circ.5}} = 1.382\underline{/37^\circ.5} = 1.096 + j0.842$$

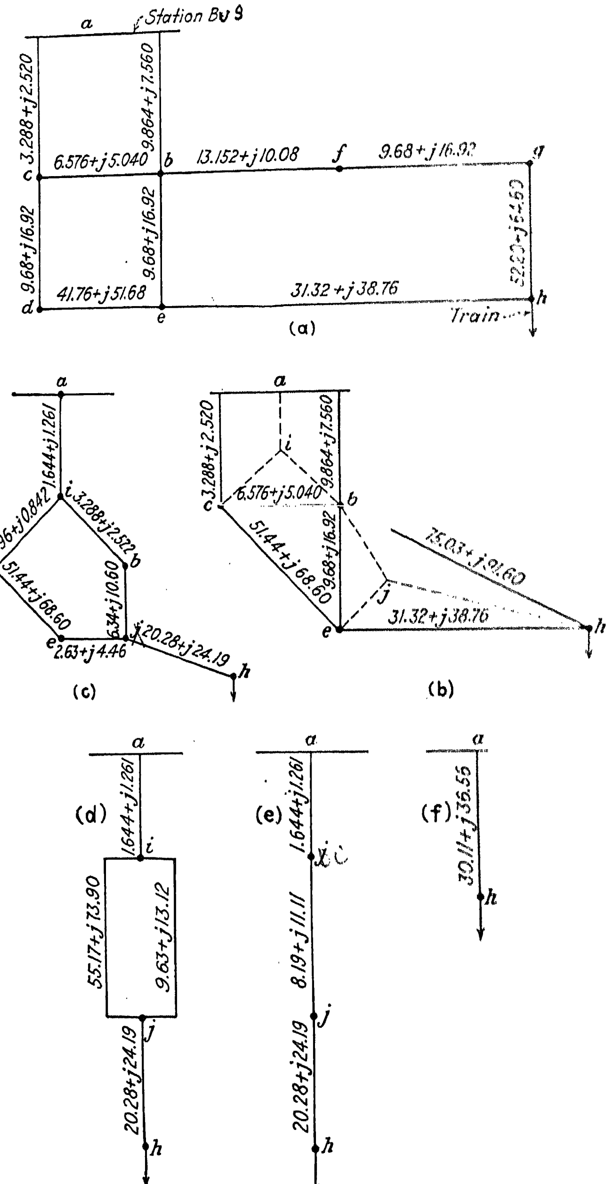


Fig. 5.—Diagrams showing successive steps in the reduction of the circuit in Fig. 4 to a single impedance between the power station and the train.

Conversion of *beh*

$$
\begin{aligned}
Z_{be} &= 9.68 + j16.92 = 19.50/\underline{60^\circ.2} \\
Z_{bh} &= 75.03 + j91.60 = 118.6/\underline{50^\circ.7} \\
Z_{eh} &= 31.32 + j38.76 = 49.77/\underline{51^\circ.1} \\
\hline
\Sigma Z &= 116.03 + j147.28 = 187.2/\underline{51^\circ.8}
\end{aligned}
$$

$$
\begin{aligned}
Z_{bj} &= \frac{19.50/\underline{60^\circ.2} \times 118.6/\underline{50^\circ.7}}{187.2/\underline{51^\circ.8}} = 12.35/\underline{59^\circ.1} \\
&\qquad = 6.34 + j10.60 \\
Z_{hj} &= \frac{118.6/\underline{50^\circ.7} \times 49.77/\underline{51^\circ.1}}{187.2/\underline{51^\circ.8}} = 31.55/\underline{50^\circ.0} \\
&\qquad = 20.28 + j24.19 \\
Z_{ej} &= \frac{49.77/\underline{51^\circ.1} \times 19.50/\underline{60^\circ.2}}{187.2/\underline{51^\circ.8}} = 5.18/\underline{59^\circ.5} \\
&\qquad = 2.63 + j4.46
\end{aligned}
$$

Using these equivalent Y circuits gives the network (Fig. 5*c*) which immediately reduces to the series-parallel circuit shown in *d*.

$$
\begin{aligned}
Z_{i(ce)j} &= 55.17 + j73.90 = 92.2/\underline{53^\circ.2} \\
Z_{i(b)j} &= 9.63 + j13.12 = 16.27/\underline{53^\circ.7} \\
\hline
\Sigma Z &= 64.80 + j87.02 = 108.7/\underline{53^\circ.3} \\
Z_{ij} &= \frac{92.2/\underline{53^\circ.2} \times 16.27/\underline{53^\circ.7}}{108.7/\underline{53^\circ.3}} = 13.80/\underline{53^\circ.6} \\
&\qquad = 8.19 + j11.11
\end{aligned}
$$

Substituting this impedance for the two parallel branches gives the circuit (Fig. 5*e*). By addition of the series impedances, the final circuit *f* is obtained. The equivalent impedance between the high-tension bus of the generating station and the train is, hence,

$$Z = 30.11 + j36.56 \text{ ohms}$$

which referred to the low-tension side becomes

$$Z = \frac{30.11 + j36.56}{4^2} = 1.882 + j2.284 \text{ ohms}$$

Current, Voltage, and Power at Train.—The total impedance Z_o referred to the low-tension side becomes

$$
\begin{aligned}
Z_o = Z + Z_t &= 1.882 + j2.284 + 16.11 + j12.09 \\
&= 17.992 + j14.374 = 23.00/\underline{38^\circ.6} \text{ ohms}
\end{aligned}
$$

Train Current:

$$I_t = \frac{V_g}{Z_o} = \frac{11{,}000/0}{23.00/\underline{38^\circ.6}} = 477.7\overline{\backslash 38^\circ.6} \text{ amp.}$$

Train Voltage:

$$
\begin{aligned}
V_t = I_t Z_t &= 477.7/\overline{38^\circ.6} \times 20.14/\underline{36^\circ.9} \\
&= 9{,}620/\overline{1^\circ.7} \text{ volts}
\end{aligned}
$$

Train Power:

$$P_t = 477.7^2 \times 16.11 \times 10^{-3} = 3{,}680 \text{ kw.}$$

Short-circuit Current Delivered by a Synchronous Machine. The currents which flow when a short circuit occurs at a point in a network will depend largely upon the general character of the short-circuit currents in the synchronous machines. When a short circuit is applied, the sustained short-circuit current will, as a rule, not be established immediately. In general, there is a transition period during which the current changes gradually from an initial to a final value (the sustained or steady-state current). The current flowing during the first instants of the transition period may easily be quite considerably greater than the sustained current.

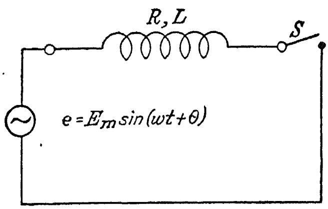


FIG. 6.—A short circuit is suddenly applied to the lumped impedance by closing the switch S.

In order to get an understanding of the character of the short-circuit currents delivered by synchronous machines, it is helpful first to review the case where a short-circuit is suddenly applied to a lumped impedance (Fig. 6) of fixed resistance and inductance at one end of which a constant sinusoidal voltage is maintained. It will be assumed that the reactance part of this impedance is predominant and that the resistance is quite small. This is a condition which would obtain in a synchronous machine.

When a constant sinusoidal voltage is impressed on a constant impedance, it is easy to obtain a rigorous mathematical solution for the current. The differential equation applying to this circuit is readily set up as follows:

$$L\frac{di}{dt} + Ri = E_m \sin(\omega t + \theta) \qquad (19)$$

As seen, this is a linear differential equation of the first order with constant coefficients, the solution of which may be expressed as the sum of two terms.[1] Referring to equation (20), which is the solution of equation (19), i_t represents the complementary

$$i = i_t + i_s \qquad (20)$$

function and i_s the particular integral. In terms of electrical quantities, the former (i_t) is equal to the transient component of the current, and the latter (i_s) is equal to the steady-state current.[2]

[1] See any standard treatise on differential equations.

[2] The general problem of transients is treated in detail in Vol. II of this treatise.

It is well known that, when the linear differential equation is of the first order, the complementary function consists of a single exponential. The exponent is found by solving the so-called *determinantal equation* obtained by equating the polynomial of derivatives to zero. In this particular case, the determinantal equation becomes

$$Lp + R = 0 \tag{21}$$

where p represents the differential operator. The exponent, hence, becomes

$$p = -\frac{R}{L} \tag{22}$$

The solution for the particular integral, or steady-state current, is readily established by applying the ordinary alternating-current steady-state theory. The complete solution may, hence, be written

$$i = A\epsilon^{-\frac{R}{L}t} + \frac{E_m}{\sqrt{R^2 + (\omega L)^2}} \sin\left(\omega t + \theta - \tan^{-1}\frac{\omega L}{R}\right) \tag{23}$$

The constant of integration A is determined by reference to the initial conditions. Assuming zero initial current, this constant becomes

$$A = -\frac{E_m}{\sqrt{R^2 + (\omega L)^2}} \sin\left(\theta - \tan^{-1}\frac{\omega L}{R}\right) \tag{24}$$

and the final solution is given by

$$i = \frac{E_m \sin\left(\theta - \tan^{-1}\frac{\omega L}{R}\right)}{\sqrt{R^2 + (\omega L)^2}} \epsilon^{-\frac{R}{L}t} + \frac{E_m}{\sqrt{R^2 + (\omega L)^2}} \sin\left(\omega t + \theta - \tan^{-1}\frac{\omega L}{R}\right) \tag{25}$$

As seen, the total current consists of a sinusoidally varying component superimposed on a component which decreases exponentially. The amplitude of the steady-state current depends only on the value of the applied voltage and the steady-state impedance of the circuit. The amplitude of the transient, on the other hand, in addition to depending on voltage and impedance, also depends on the point on the voltage wave at which the switch is closed, *i.e.*, it depends on the angle θ. Assuming that the resistance is small, so that the angle of lag of the steady-state current behind the voltage is nearly 90 deg., it is seen that the

transient will entirely disappear when the switch is closed (*i.e.*, short circuit occurs) as the voltage wave passes through one of its maximum values ($\theta = 90$ or 270 deg.). If, on the other hand, the switch is closed on zero voltage ($\theta = 0$ or 180 deg.), the initial amplitude of the transient is equal and opposite to the maximum amplitude of the steady-state current. In this case, therefore,

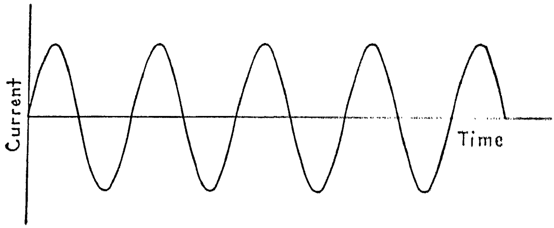


FIG. 7.—The curve shows the steady-state current which will be set up immediately in the circuit in Fig. 6 when the switch is closed as the applied voltage passes through one of its maxima.

the maximum possible total current will be reached after the elapse of a time corresponding to half a cycle.

Figures 7 and 8 illustrate the currents in the two cases. In the former, the steady-state current is set up immediately. In the latter, the wave of total current is a completely "offset" wave obtained by adding the exponential transient and the steady-state

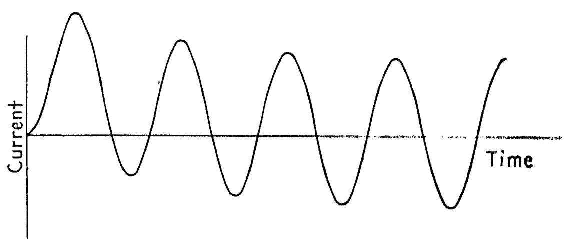


FIG. 8.—The curve shows the current which will flow in the circuit in Fig. 6 when the switch is closed as the applied voltage passes through zero. Mathematically this curve consists of a sinusoid superimposed on an exponential.

sinusoidal current. If the short circuit occurs at values of voltage intermediate between zero and maximum, a "partly offset" curve of total current evidently results.

If the reactance of the synchronous machine[1] were a constant quantity, the character of the transients produced by a short circuit (polyphase or single-phase) applied at the machine

[1] DOHERTY, R. E., and O. E. SHIRLEY, "Reactance of Synchronous Machines and its Applications," *Trans. A.I.E.E.*, p. 1209, 1918.

terminals would be as discussed above, with the exception that, mathematically, the transients would consist of two exponentials instead of merely one. This is due to the inductive coupling between the field and armature circuits. On account of the change in reactance, however, matters become more complicated, and a rigorous mathematical solution is not readily obtained.[1]

When a short circuit is applied at such an instant that the transient disappears, the total current will have the shape illustrated in Fig. 9. As seen, this curve is not offset from the zero axis and may be looked upon as being symmetrical with respect to the latter. It is furthermore seen that the amplitudes decrease as time elapses. This decrease is caused by the fact that the reactance increases from a low initial value to a much higher value

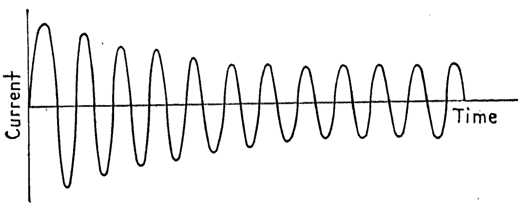


FIG. 9.—Symmetrical short-circuit current delivered by an alternator. Short circuit applied as the voltage wave passes through a maximum value.

for the steady state. Since the flux in the magnetic circuit of the machine cannot change abruptly when the short circuit is applied, the initial current will be limited by the total leakage reactance of the machine. This total leakage reactance is usually called the *transient reactance* and is, in machines without damper windings, equal to the sum of the leakage reactance of the armature and the leakage reactance of the field. If damper windings are present, their effect must be taken into account and the value of the transient reactance modified. Furthermore, eddy currents set up in the field structure will also affect the value of the transient reactance. In general, the armature leakage reactance will constitute the major part of the transient reactance. If the exact value of the transient reactance, therefore, is not known, the armature leakage reactance may be used in place of the transient reactance. As a rule, this will not give rise to appreciable errors

[1] Probably the most up-to-date and rigorous treatment of short-circuit currents in synchronous machines is given in Part IV of a recent paper by R. E. DOHERTY and C. A. NICKLE: "Synchronous Machines, IV. Single-phase Short Circuits," *Jour. A.I.E.E.*, 1928.

except, perhaps, in salient-pole machines without dampers. Furthermore, the discrepancies, if any, will be in a conservative direction, since a reactance which is slightly too low is used.

As time elapses, the armature reaction builds up, and the net flux in the magnetic circuit decreases. The effect of this is equivalent to a gradual increase in the reactance of the machine.

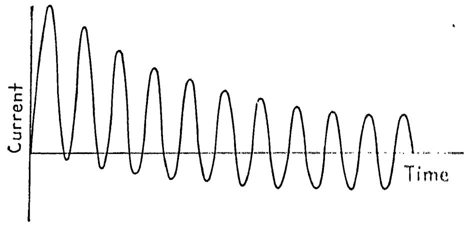


FIG. 10.—Dissymmetrical short-circuit current delivered by an alternator. This curve is completely offset and is obtained when the short circuit is applied as the voltage wave passes through zero.

The reactance finally becomes equal to the synchronous reactance, and steady-state conditions are obtained.

If the short circuit is applied in such a manner that the transient or exponential terms do not disappear, the total current wave will be offset from the horizontal axis and, hence, be dissymmetrical. The total wave may be looked upon as consisting

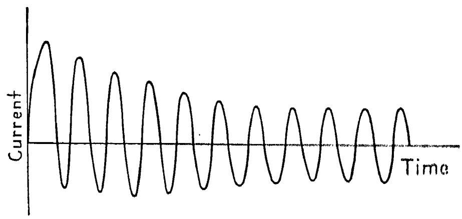


FIG. 11.—Dissymmetrical short-circuit current delivered by an alternator. This curve is not completely offset and is obtained when the short circuit is applied when the voltage has a value between zero and maximum.

of a symmetrical wave of the type shown in Fig. 9 superimposed on the transient components. If the short circuit is applied as the voltage of the phase involved passes through zero, a completely offset wave is obtained, as illustrated in Fig. 10. If the short circuit occurs when the voltage has a value between zero and maximum, a partially offset wave will result, as shown in Fig. 11.

Although the complete story of a short-circuit current is given nly when the actual instantaneous values of current are known t various times, it has become standardized practice in practical hort-circuit calculations to use effective or root-mean-square alues exclusively. The total dissymmetrical wave of current is ooked upon as being made up of a direct-current component,

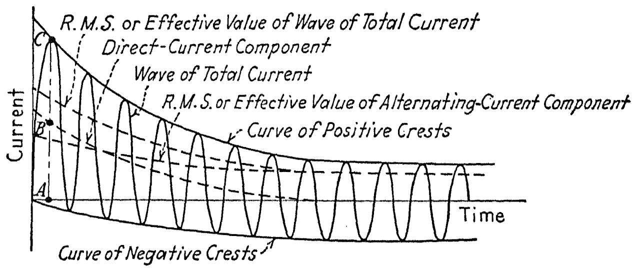


FIG. 12.—This diagram shows how the direct-current and the alternating-current component in a dissymmetrical wave of short-circuit current are determined.

which decreases with time and eventually disappears, and an lternating-current component, the amplitude of which also lecreases as time elapses but which finally reaches a steady state. When the total wave of current, as shown, for instance, in Fig. 12, is known, the direct-current component for any value of time nay be determined by drawing smooth curves through the maximum positive and negative amplitudes. These curves are

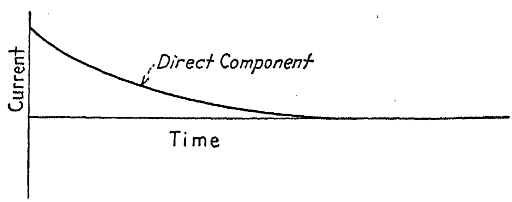


FIG. 13.—Direct-current component in the current wave in Fig. 12.

termed *curves of positive crests* and *negative crests*, respectively. By halving the vertical distance between the curves of positive and negative crests, the amplitude of the direct-current component is obtained, as indicated in Fig. 12, and also shown separately in Fig. 13. Each loop of the total current when considered with respect to the direct-current component is then assumed to represent one half-cycle of a sinusoidal current. Obviously this

is an approximation. The effective value of each of these half-cycles is calculated by dividing the maximum amplitudes with respect to the direct-current component by $\sqrt{2}$. Considering the first maximum in Fig. 12, for instance, the direct-current component is given by

$$I_{\text{d.c.}} = AB \tag{26}$$

and the effective value of the alternating-current component by

$$I_{\text{a.c.}} = \frac{BC}{\sqrt{2}} \tag{27}$$

The alternating-current component plotted alone will give a curve of the type shown in Fig. 9. It is likewise shown in Fig. 14, where a curve of the effective value also has been drawn in. The effective value of the total current is obtained by combining the

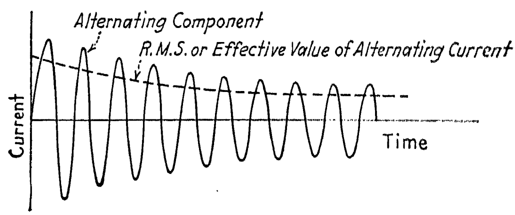


FIG. 14.—Alternating-current component in the wave of short-circuit current in Fig. 12. Both instantaneous and effective values are shown.

direct-current component with the effective value of the alternating-current component for the various values of time. Obviously, the effective value of the total current is equal to the square root of the sum of the squares of the effective values of the components. The combination is, hence, carried out in accordance with the following equation

$$I_{\text{eff.}} = \sqrt{I^2_{\text{d.c.}} + I^2_{\text{a.c.}}} \tag{28}$$

Decrement Factors.—The character of the short-circuit current which will be delivered by a synchronous machine when a short circuit is applied at its terminals has been briefly described above. The short-circuit currents which will flow in a network into which one or more generators feed will be of the same general type. On account of the impedance which the network itself offers, these short-circuit currents will obviously be lower than if the short circuit were applied at the terminals of the machines.

In order to obtain data which might be applicable to general short-circuit calculations, extensive tests have been performed by

e manufacturers, particularly by the General Electric Company ıd the Westinghouse Electric and Manufacturing Company. hese tests comprised the determination of short-circuit currents livered by machines of various ratings and designs for three neral types of short circuit, namely,

1. Symmetrical three-phase short circuit.
2. Single-phase line-to-line short circuit.
3. Line-to-neutral short circuit.

The short circuits were applied directly at the terminals of the achines and also with external reactance between the machines ıd the short circuit. The external reactance was varied so that range of total reactance, *i.e.*, external reactance plus machine actance, between a very low value and 100 per cent, was coved. Reactances up to 15 per cent were inside the machines, ıd, for higher values, 15 per cent were inside the machines and ıe remainder external to the machines.

The excitation used in all tests was that corresponding to full ad at 80 per cent power factor (lagging). It has been previously

ABLE I.—SYSTEM SHORT-CIRCUIT CURRENT FACTORS APPLICABLE TO THREE-PHASE SHORT CIRCUITS ON THREE-PHASE SYSTEMS

Time rom stant of hort rcuit, conds	Root-mean-square total current expressed in number of times full-load current for various per cent reactance											
	5 per cent	8 per cent	10 per cent	12 per cent	15 per cent	20 per cent	30 per cent	40 per cent	50 per cent	60 per cent	75 per cent	100 per cent
0.00	35.00	22.00	17.75	14.90	12.00	9.01	6.00	4.52	3.55	2.94	2.36	1.74
0.05	21.18	13.60	11.10	9.40	7.74	5.89	3.98	3.04	2.41	2.03	1.64	1.23
0.08	18.15	11.65	9.50	8.15	6.72	5.14	3.50	2.89	2.15	1.81	1.47	1.11
0.10	16.50	10.70	8.81	7.52	6.22	4.79	3.28	2.54	2.03	1.72	1.40	1.06
0.15	13.48	8.85	7.36	6.32	5.30	4.13	2.87	2.25	1.83	1.56	1.28	0.981
0.20	11.90	7.86	6.56	5.66	4.82	3.74	2.67	2.11	1.72	1.48	1.22	0.943
0.25	10.54	7.10	6.00	5.20	4.45	3.53	2.52	2.01	1.66	1.42	1.18	0.919
0.30	9.56	6.50	5.55	4.85	4.19	3.35	2.42	1.94	1.61	1.39	1.16	0.904
0.40	8.33	5.80	4.96	4.38	3.83	3.10	2.28	1.86	1.55	1.35	1.13	0.888
0.50	7.30	5.15	4.48	3.99	3.52	2.91	2.18	1.79	1.51	1.32	1.11	0.877
0.70	5.94	4.35	3.84	3.48	3.13	2.64	2.04	1.70	1.45	1.27	1.08	0.862
1.00	4.60	3.55	3.24	2.98	2.75	2.38	1.90	1.61	1.39	1.23	1.05	0.843
1.50	3.42	2.90	2.70	2.56	2.43	2.17	1.78	1.54	1.34	1.19	1.03	0.836
2.00	2.72	2.43	2.34	2.27	2.21	2.02	1.71	1.49	1.31	1.17	1.02	0.828
3.00	2.00	2.00	2.00	2.00	2.00	1.88	1.63	1.44	1.28	1.15	1.00	0.820

stated that the magnitude of the short-circuit current depends largely on the value of voltage at the instant the short circuit occurs. In order to be on the conservative side, it is advisable to figure with the maximum possible values. In all these tests, therefore, whether symmetrical three-phase or single-phase, the short circuit was applied in such a manner that maximum possible current was obtained in at least one phase. Oscillograms of this current were taken and analyzed for their root-mean-square values of current in the manner previously described. In this way, it was possible to plot a series of curves of root-mean-square values of short-circuit current *versus* time for different values of reactance.

Different types and sizes of machines would obviously give somewhat different values of short-circuit current. The tests results, however, were compared by the manufacturers responsible for the tests, averaged up, and a set of values selected which was considered representative of the short-circuit currents which machines of average design would deliver. These data in the

TABLE II.—SYSTEM SHORT-CIRCUIT CURRENT FACTORS APPLICABLE TO SINGLE-PHASE LINE-TO-LINE SHORT-CIRCUITS ON THREE-PHASE SYSTEMS

Time from instant of short circuit, seconds	Root-mean-square total current expressed in number of times full-load current for various per cent reactance											
	5 per cent	8 per cent	10 per cent	12 per cent	15 per cent	20 per cent	30 per cent	40 per cent	50 per cent	60 per cent	75 per cent	100 per cent
0.00	35.00	22.02	17.82	14.88	12.00	9.01	6.00	4.52	3.51	2.95	2.36	[illegible]
0.05	21.80	14.00	11.46	9.69	7.95	6.07	4.12	3.15	2.49	2.08	1.67	[illegible]
0.08	18.53	12.02	9.90	8.43	7.01	5.38	3.69	2.84	2.25	1.90	1.51	1.11
0.10	16.93	11.10	9.18	7.85	6.56	5.07	3.50	2.70	2.16	1.80	1.44	1.06
0.15	13.92	9.32	7.80	6.75	5.87	4.50	3.16	2.47	1.99	1.67	[illegible]	0.98
0.20	12.30	8.36	7.09	6.19	5.31	4.21	3.00	2.36	1.90	1.60	1.29	0.94
0.25	11.08	7.66	6.55	5.76	5.00	4.00	2.89	2.29	1.86	1.56	1.26	0.92
0.30	10.18	7.15	6.15	5.45	4.79	3.86	2.82	2.25	1.82	1.54	1.24	0.90
0.40	8.96	6.45	5.62	5.04	4.48	3.67	2.73	2.19	1.79	1.51	1.21	0.89
0.50	8.01	5.89	5.20	4.71	4.24	3.51	2.66	2.15	1.77	1.49	1.20	0.87
0.70	6.73	5.15	4.63	4.27	3.92	3.31	2.57	2.10	1.73	1.47	1.18	0.86
1.00	5.46	4.42	4.08	3.84	3.61	3.12	2.48	2.05	1.70	1.45	1.16	[illegible]
1.50	4.41	3.81	3.62	3.48	3.35	2.95	2.40	2.01	1.68	[illegible]	1.15	[illegible]
2.00	3.69	3.40	3.30	3.23	3.17	2.84	2.35	1.98	1.66	[illegible]	[illegible]	[illegible]
3.00	3.00	3.00	3.00	3.00	3.00	2.73	2.30	1.95	1.64	1.40	1.13	0.81

form of tables and curves[1] are now available for the engineer who has to perform calculations of short-circuit currents in networks.

Instead of giving actual currents in amperes and reactances in ohms, the former are given in terms of rated current of the machine involved and the latter in per cent based on the rating. In this manner, the data are immediately made applicable to all sizes of machines. The factors which indicate the number of times rated current which will flow are called *decrement factors.* Tables I, II, and III give the decrement factors for symmetrical three-phase short circuits, single-phase line-to-line short circuits, and single-phase line-to-neutral short circuits, respectively.

It should be carefully noted that Table III of decrement factors for single-phase line-to-neutral short circuits is of limited appli-

TABLE III.—SYSTEM SHORT-CIRCUIT CURRENT FACTORS APPLICABLE TO SINGLE-PHASE LINE-TO-NEUTRAL SHORT-CIRCUITS ON THREE-PHASE SYSTEMS[1]

Time from instant of short circuit, seconds	Root-mean-square total current expressed in number of times full-load current for various per cent reactance.											
	5 per cent	8 per cent	10 per cent	12 per cent	15 per cent	20 per cent	30 per cent	40 per cent	50 per cent	60 per cent	75 per cent	100 per cent
0.00	52.90	33.40	27.10	22.70	19.30	12.80	7.67	5.49	4.05	3.28	2.54	1.85
0.05	33.00	21.25	17.48	14.80	12.80	8.75	5.39	3.91	2.93	2.38	1.85	1.35
0.08	28.50	18.40	15.20	12.89	11.30	7.85	4.85	3.55	2.68	2.20	1.70	1.23
0.10	25.70	16.90	14.05	12.06	10.52	7.38	4.65	3.42	2.58	2.10	1.64	1.19
0.15	21.30	14.40	12.10	10.50	9.35	6.65	4.29	3.20	2.41	1.98	1.52	1.09
0.20	18.80	12.87	10.95	9.60	8.58	6.25	4.10	3.07	2.34	1.91	1.49	1.07
0.25	17.10	11.90	10.10	9.10	8.10	6.00	3.99	3.01	2.30	1.89	1.47	1.05
0.30	15.80	11.10	9.50	8.60	7.80	5.85	3.95	2.96	2.29	1.87	1.46	1.05
0.40	13.90	10.06	8.83	7.96	7.28	5.59	3.86	2.94	2.28	1.86	1.45	1.03
0.50	12.60	9.32	8.20	7.50	6.89	5.46	3.80	2.91	2.26	1.85	1.45	1.02
0.70	10.60	8.19	7.42	6.88	6.46	5.18	3.73	2.87	2.25	1.85	1.44	1.02
1.00	8.71	7.14	6.63	6.22	6.00	4.95	3.66	2.84	2.24	1.84	1.42	1.01
1.50	7.17	6.27	5.98	5.77	5.61	4.76	3.60	2.81	2.23	1.84	1.41	1.01
2.00	6.11	5.67	5.53	5.43	5.35	4.67	3.56	2.80	2.23	1.83	1.40	1.00
3.00	5.10	5.10	5.10	5.10	5.10	4.51	3.52	2.78	2.22	1.83	1.40	1.00

[1] Note the limited applicability of the factors in this table. See statement on p. 22.

[1] Some of the manufacturers' publications contain this information. See also "Relay Handbook," published by the National Electric Light Association, New York, 1926, and the paper "The Application of Decrement Factors in Short-circuit Studies," by W. R. WOODWARD, *Elec. Jour.* p. 213, 1924.

cation in practice. It can be used only with Y-connected generators and then only when (*a*) the neutral points of all generators are solidly grounded and distribution takes place at generator voltage or (*b*) all transformer connections are Y-Y with secondary neutral points solidly grounded and with primary and generator neutral points interconnected. There are but few systems in operation which would fully conform with these specifications as to connections and grounding.

Application of Decrement Factors.—Practical short-circuit calculations making use of the experimentally determined decrement factors involve, as already stated, several assumptions. At this point, these may appropriately be summarized, as follows:

1. Transient characteristics of generators of normal design.
2. The effect of resistance, leakance, and capacitance is neglected.
3. The impedance at the point of short circuit is zero.
4. The excitation of the generators corresponds to full load at 80 per cent power factor (lagging).
5. The short circuit is established at the point of the voltage wave giving maximum possible instantaneous current.
6. There are no voltage regulators.
7. All reactance values up to and including 15 per cent are inside the generators and, for higher values, 15 per cent inside the generators and the remainder external to the generators.

If a single synchronous machine supplies a short circuit through an external reactance and it is desired to determine the amount of current which flows after the elapse of a certain time (t_0), the procedure will obviously be as follows:[1]

The total reactance to the point of short circuit is first determined. This reactance equals the sum of the external reactance and the transient reactance of the machine. In adding these reactances, they are both expressed in per cent, preferably on a base corresponding to the kilovolt-ampere rating of the machine.

The proper table of decrement factors is entered with the percentage total reactance and the time at which the short-circuit current is desired, and the decrement factor selected. The rated current of the machine times the decrement factor is then equal to the root-mean-square value of the short-circuit current which will flow at the elapse of the specified time.

[1] WOODWARD, W. R.: "The Application of Decrement Factors in Short-circuit Studies," *loc. cit.* See also "Relay Handbook," *loc. cit.*

If it is desired to determine the initial symmetrical value of the short-circuit current, this can be obtained by dividing the rated current by the percentage reactance and multiplying by 100. Hence,

$$I_0 = \frac{I_{\text{rated}} \times 100}{X_0} \tag{29}$$

where I_0 represents the initial symmetrical short-circuit current and X_0 the total reactance to the point of short circuit. That the equation is correct may be appreciated from the following consideration: If the reactance were 100 per cent, evidently rated current would flow, assuming that the internal voltage of the machine has its rated value. When the reactance is different from 100 per cent and the internal voltage normal, the ratio of the current which actually flows and the rated current must obviously equal the inverse ratio of the actual reactance and the 100 per cent reactance.

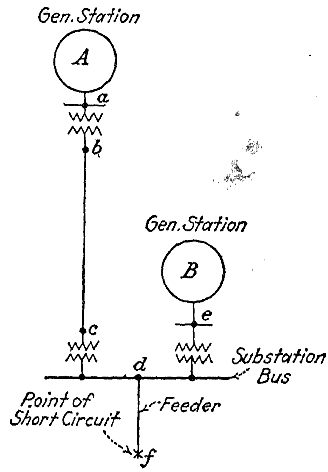


FIG. 15.—Simple two-station system used in discussing application of decrement factors.

When a short circuit occurs on a network, the short-circuit current will, as a rule, be supplied from several generating stations.[1] There are two methods of applying the decrement factors in such cases. Either a single decrement factor may be applied to the synchronous machines lumped, or else a separate decrement factor may be applied to each station. In order to illustrate these methods more fully, the layout shown in Fig. 15 will be considered. This layout involves two generating

[1] If, in one or more of the stations or at other points of the system, there are large synchronous motors or condensers operating, these machines may be treated as generators for the first few seconds after the occurrence of the short circuit. Owing to their inertia, they will continue to run at synchronous speed for a short time and will, during this period, supply current to the short circuit as if they were operating as generators. After a while, these machines will slow down and eventually come to rest. Much before this happens, however, their own circuit breakers may have opened and disconnected them from the system. In no event, therefore, will the synchronous motors or condensers contribute to the sustained short-circuit currents.

stations only but serves to illustrate the principles, since exactly the same methods apply when any number of stations supply the network. In Fig. 15, a short circuit occurs on a feeder a certain distance from the bus of a substation. Generating station A supplies power to the substation over a high-tension transmission line with step-up and step-down transformers. Station B, located near the substation, supplies power to the latter over a bank of step-up transformers.

Figure 16 shows a diagram of the reactances involved. Both stations are connected to a common hypothetical bus at which the normal voltage of the short-circuited feeder is assumed to be maintained. The reactances Aa and Be represent the transient reactances of the two stations. The reactances ab, cd, and ed represent the three banks of transformers; the reactance bc the transmission line; and the reactance df the feeder between the substation bus and the point of short circuit.

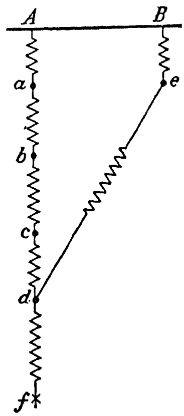


FIG. 16.—Reactance diagram of the system in Fig. 15.

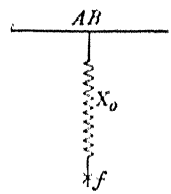


FIG. 17.—Equivalent reactance of the system in Fig. 15.

These reactances should all be expressed in per cent on a common kilovolt-ampere base. The selected kilovolt-ampere base may be taken as the rating of either generating station, as the combined rating of the two stations, or be selected arbitrarily. When the layout involves several generating stations, an arbitrary base of suitable size is most commonly used.

The network reactances are next combined so that the circuit is reduced to a single reactance between the hypothetical bus and the point of short circuit, as shown in Fig. 17. This process of reduction must always be gone through independent of the method of application of decrement factors used.

Method 1. Application of Decrement Factors to the System Lumped.—It will be assumed that the reactances have been cal-

ılated on an arbitrary kilovolt-ampere base different from the ,ting of either station and also different from the combined rat-.gs of the stations. Let the total reactance on this base be X_0. If a single decrement factor is to be applied to the two stations ›mbined, the per cent total reactance must be converted to a ıse equal to the combined capacity of the two stations. When ıe voltage is unchanged, the per cent reactance is directly pro-ɔrtional to the kilovolt-ampere base. Hence, the per cent :actance based on the total rating of the machines is given by

$$X'_0 = X_0 \frac{\text{Rating of machines}}{\text{Base kilovolt-ampere}}$$

$$= X_0 \frac{A + B}{\text{Base}} \tag{30}$$

The proper table or curve is now entered with the reactance $'_0$ and the decrement factor (k) corresponding to the desired me (t_0) selected. The short-circuit current which will flow at ıe elapse of the time t_0 is then given by

$$I_{s.c.} = kI_{\text{rated}} \tag{31}$$

Method 2. Application of Decrement Factors to Each Generat-ıg Station Separately.—Also, in this case, let the total reactance alculated on some arbitrary base be X_0. In order to determine he proper decrement factor which should be used for each tation, it is necessary first to calculate the initial symmetrical urrent in the short circuit by equation (27). All stations ontribute their share to this short-circuit current. The amount ʼhich each station supplies is next determined by properly pportioning the total current (I_0) between the stations. In the articular case under discussion, the initial symmetrical short-ircuit currents supplied by the stations A and B, respectively, .re given by

$$I_{0(A)} = I_0 \frac{X_{Bd}}{X_{Ad} + X_{Bd}} \tag{32}$$

$$I_{0(B)} = I_0 \frac{X_{Ad}}{X_{Ad} + X_{Bd}} \tag{33}$$

Making use of the initial symmetrical short-circuit currents, an quivalent reactance between each station and the point of short ircuit is calculated as follows:

$$X_{0(A)} = \frac{I_{\text{rated}(A)} \times 100}{I_{0(A)}} \tag{34}$$

$$X_{0(B)} = \frac{I_{\text{rated}(B)} \times 100}{I_{0(B)}} \tag{35}$$

Since these reactances are computed by means of the rated currents of the machines, they are obtained on the kilovolt-ampere base of the respective machines. The tables or curves of decrement factors may, therefore, be entered with these reactances and a separate decrement factor selected for each station at the desired time. The short-circuit current which each station delivers after the elapse of this time is then obtained by multiplying the rated current of that particular station by its decrement factor. Letting k_A and k_B represent the decrement factors of the two stations, the short-circuit currents at the time t_0 are given by

$$I_{s.c.(A)} = k_A I_{\text{rated}(A)} \qquad (36)$$

$$I_{s.c.(B)} = k_B I_{\text{rated}(B)} \qquad (37)$$

The total current flowing in the short circuit is obviously given by the sum of the currents supplied by each machine. Hence,

$$I_{s.c.} = I_{s.c.(A)} + I_{s.c.(B)} \qquad (38)$$

In carrying out calculations of this type, it is convenient, particularly when the system involves several generating stations, to arrange the calculations in tabular form. A suitable scheme is suggested in Table IV.

TABLE IV.—SHORT-CIRCUIT CALCULATIONS

Station	Amperes at normal voltage		Per cent equivalent reactance	At time = t_0 seconds	
	Rated	Initial symmetrical		Decrement factor	Short-circuit amperes
A............					
B............					

Evidently, of the two methods described above for the application of decrement factors, the first one in which the decrements are applied to the system lumped is the easier one. The second one involves a good deal more labor. The question then arises which one is preferable in a practical case.

Neither method is rigorous even if the general assumptions on which these short-circuit calculations rest are disregarded. When the single decrement factor is applied to all stations combined, the fact that large and small stations, in general, have widely different decrements is not properly taken into account. Hence,

accuracies are obviously introduced by lumping the stations. on the other hand, a separate factor is applied to each station, e voltage drops in the various branches of the circuit may not lance up at all values of time. Inaccuracies may consequently anticipated also with this method.

It is difficult to predict, in general, which method will give the tter results. It may be said that, as a rule, the first method ll give slightly higher values of currents than the second. om this standpoint, the former is preferable in that conserva- re results are obtained. Although this will be so in most cases, may not be universally true. In cases where a large station of w reactance is located in the immediate neighborhood of the int of short circuit, currents calculated by the second method ay actually be larger than those calculated by the first method, rticularly at small values of time. Ordinarily, however, the o methods give results that are very close together. This may seen, for instance, by comparing the results in Example 2 tained by computations based on both methods.

Since, as already stated, the first method is the easier and orter one, it is also the one which is most frequently used in actice.

Solutions by Calculating Table.—In spite of the simplifying sumptions, calculations of short-circuit currents in complicated tworks require considerable time and labor. This is obviously great disadvantage where a frequent check-up on short-circuit rrents is necessary, which is the case whenever new extensions d load additions to a system or installations of new circuit eakers and additional equipment are planned.

In order to facilitate determination of short-circuit currents, lculating tables[1] have been used quite extensively. These

[1] Lewis, W. W., "Calculation of Short-circuit Currents in Alternating- rrent Systems," *Gen. Elec. Rev.*, p. 140, 1919.

Woodward, W. R., R. D. Evans, and C. L. Fortescue, "Calculating ort-circuit Currents in Networks," *loc. cit.* Part I of this article (by oodward) discusses testing with miniature networks.

Lewis, W. W., "A New Short-circuit Calculating Table," *Gen. Elec. v.*, p. 669, 1920.

Corbett, L. J., "A Short-circuit Calculating Table," *Elec. World*, p. 985, 22.

Dillard, E. W., "A Short-circuit Calculating Table," *Elec. World*, 797, 1923.

Schurig, O. R., "Experimental Determination of Short-circuit Currents Electric Power Networks," *Trans. A.I.E.E.*, p. 10, 1923.

consist of a combination of resistances connected so as to represent the network under consideration. Since in practical short-circuit calculations reactances only are considered, it is possible to represent these on the calculating table by pure resistances and to use direct current. At points where generating stations feed into the actual network, the proper direct-current voltages are impressed on the miniature system. The currents are then simply recorded by direct-current ammeters.

When a calculating table of this type is used, it is obviously possible to obtain values of current for any number of simultaneous short circuits, if this is desired. Ordinarily, however, a short circuit occurs at one point only at a time, so that, as a rule, a single short circuit is all that has to be considered. The currents measured will represent either initial symmetrical short-circuit currents or else sustained short-circuit currents. It all depends upon the values of resistance used to represent the generator reactances. If values corresponding to transient reactances are used, the initial symmetrical short-circuit currents will be measured. If, on the other hand, the generator resistances are adjusted to correspond to synchronous reactances, sustained short-circuit currents will be obtained. It is thus seen that the calculating table does not give the solution at the elapse of a certain time interval. If this is required, decrement factors must be applied, as in the ordinary analytical solution. The equivalent reactance (or reactances), however, with which the decrement tables are entered can be determined by means of the values of current read on the calculating table.

Sometimes the calculating table is permanently set up to represent a specific system. This is the simplest and involves resistances of fixed values only. Many operating companies who use calculating tables for short-circuit determination have tables of this type. As their systems are extended, they simply add the necessary fixed resistances on the calculating table in order to keep the miniature system up to date.

It is obviously also possible to construct calculating tables with a number of variable resistance units. By a convenient plug board or dial arrangement, these resistances may be combined and connected so as to represent any arbitrary system or network within the range of the table. Such calculating tables are flexible and, hence, useful in cases where determination of short-circuit currents is not confined to any specific system. Some of the

manufacturers and consulting-engineering offices have calculating tables of this type.

Recently it has been proposed to extend the use of calculating tables also to the solution of networks under normal operating conditions.[1] In order to make this possible, the miniature system must be built up of impedance units instead of merely resistances, and alternating currents must be used instead of direct currents. The loads in such a system are represented by impedances. The generating stations and other synchronous machines, such as motors and condensers, cannot, in this case, be simulated merely by impressing an alternating voltage of the correct magnitude. It must be ascertained that the voltages also have the correct phase displacement relative to each other. This can be done by using phase shifters[2] to represent the synchronous machines. By these phase shifters, voltages of the correct magnitude as well as phase displacement are obtained.

In addition to being useful for the determination of voltages, currents, and power under normal conditions, the alternating-current calculating table will obviously also give the solution of short-circuit problems. It should actually be superior to the direct-current table for the latter purpose, since it makes it possible also to take resistance into account. Furthermore, its use eliminates the necessity of the assumption that all generator voltages are in phase.

EXAMPLE 2

Statement of Problem

Four generating stations A, B, C, and D feed into a transmission network, as shown in the sketch. The nominal voltages of the various parts of the system, as well as the reactances of the interconnecting lines, are indicated on the diagram, Fig. 18. All reactances given are on a 20,000-kv.-a. base.

Assuming a short circuit on one of the 22-kv. feeders in power station A, it is desired to determine the amount of current that the switch S will have to interrupt after a time of 0.2 sec.

---

[1] SCHURIG, O. R., "The Solution of Electric Power Transmission Problems in the Laboratory by Miniature Circuits," *Gen. Elec. Rev.*, p. 611, 1923.

———, "A Miniature Alternating-current Transmission System for Network and Transmission-system Problems," *Trans. A.I.E.E.*, p. 831, 1923.

SPENCER, H. H., and H. L. HAZEN, "Artificial Representation of Power Systems," *Trans. A.I.E.E.* p. 72, 1925.

[2] Described in the paper by Spencer and Hazen, *loc. cit.*

Equipment in Station A:

Two 5,000-kv.-a. generators with 18 per cent reactance.

One 7,812-kv.-a. generator with 25.6 per cent reactance.

One 7,500-kv.-a. synchronous condenser with 85.4 per cent reactance.

The two-circuit transformer in this station has 6.3 per cent reactance, and each of the three-circuit transformers has 5.7 per cent reactance between the 66- and 22-kv. windings; 6.3 per cent reactance between the 66- and 4-kv. windings; and 5.7 per cent reactance between the 22- and 4-kv. windings.

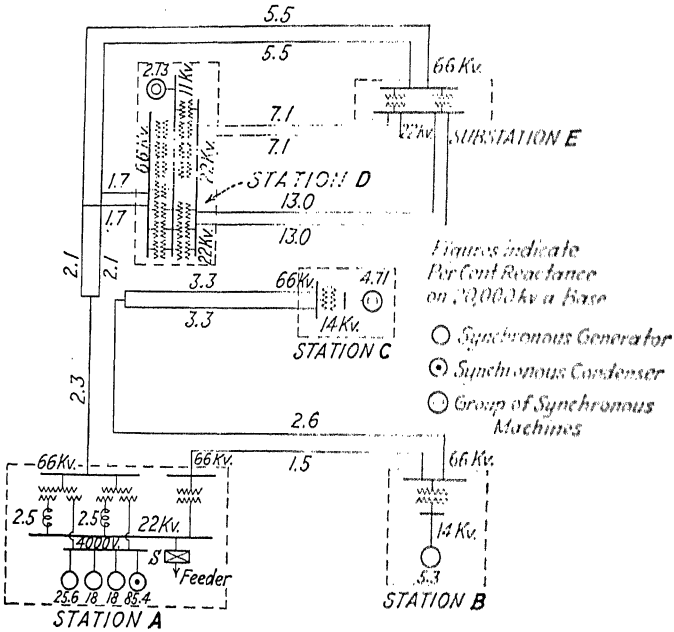


FIG. 18.—Single-line diagram showing layout of power system containing four generating stations and a substation. The short-circuit calculations in Example 2 are based on this layout.

Equipment in Station B:

One 37,500-kv.-a. synchronous generator with 5.3 per cent reactance.

The transformer in this station has 4.4 per cent reactance.

Equipment in Station C:

Total capacity of synchronous machinery 47,050 kv.-a. with reactance of 4.71 per cent.

The transformers in this station have 5.4 per cent reactance.

Equipment in Station D:

Total capacity of synchronous machinery 134,500 kv.-a. with combined reactance of 2.13 per cent.

Four of the seven 66/11-kv. transformers have 58.6 per cent reactance each, two have 19.8 per cent reactance each, and one has 12.9 per cent react-

ance. The seven $^{22}\!/_{11}$-kv. transformers are identical, each having 24 per cent reactance.

Equipment in Substation E:

The two $^{66}\!/_{22}$-kv. transformers in this station have 13 per cent reactance each.

The various loads connected to the system have not been indicated, as all load currents are to be neglected as compared to the short-circuit currents. The magnitude and phase of the voltages at the generating stations will be considered the same; in other words, all generators may be assumed connected to one common bus.

Solution

Total reactance of synchronous machines in station A

$$\frac{1}{\frac{1}{25.6} + \frac{2}{18} + \frac{1}{85.4}} = 6.18 \text{ per cent}$$

Each of the three-circuit transformers in this station may be replaced by an equivalent Y-connected network (see Chap. II, equations (70), (71), and (72)). Designating the 66-kv. winding as No. 1, the 22-kv. winding as No. 2, and the 4,000-volt winding as No. 3, the reactances to be assigned to the branches of the equivalent Y become

$$Z_1 = \frac{5.7 + 6.3 - 5.7}{2} = 3.15 \text{ per cent}$$

$$Z_2 = \frac{5.7 + 5.7 - 6.3}{2} = 2.55 \text{ per cent}$$

$$Z_3 = \frac{6.3 + 5.7 - 5.7}{2} = 3.15 \text{ per cent}$$

Total reactance of the seven $^{66}\!/_{11}$-kv. transformers in station D

$$\frac{1}{\frac{4}{58.6} + \frac{2}{19.8} + \frac{1}{12.9}} = 4.05 \text{ per cent}$$

Total reactance of the group of four $^{22}\!/_{11}$-kv. transformers in station D

$$^{24}\!/_{4} = 6.0 \text{ per cent}$$

Total reactance of the group of three $^{22}\!/_{11}$-kv. transformers in station D

$$^{24}\!/_{3} = 8.0 \text{ per cent}$$

Using the values calculated above for machine and transformer combinations in conjunction with the rest of the data given on the layout (Fig. 18), the circuit diagram (Fig. 19*a*) results. By going through the steps indicated in Fig. 19*a* to *g*, the circuit is reduced to a single reactance between a hypothetical 22-kv. bus and the point of short circuit. This reactance is 4.63 per cent.

$$\text{Base current} = \frac{20{,}000}{\sqrt{3} \times 22} = 525 \text{ amp.}$$

Initial symmetrical short-circuit current corresponding to 4.63 per cent reactance

$$I_0 = \frac{525 \times 100}{4.63} = 11{,}340 \text{ amp.}$$

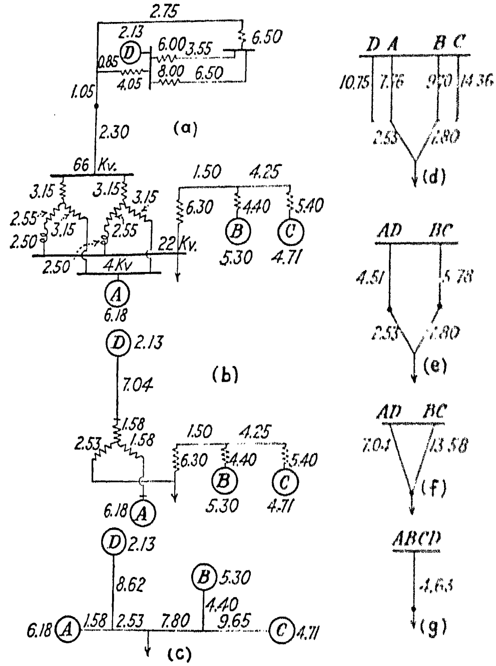


FIG. 19.—Diagrams showing successive steps in reducing the system in Fig. 18 to a single equivalent reactance between the hypothetical bus and the point of short circuit.

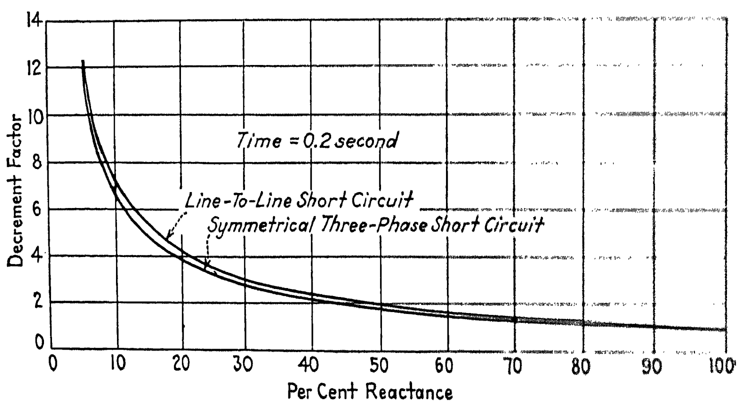


FIG. 20.—Decrement factors for symmetrical three-phase short circuits and single-phase line-to-line short circuits plotted *versus* reactance for a definite time $t_0 = 0.2$ sec. Data for these curves have been obtained from Tables I and II.

1. *Application of Decrements to Each Generating Station Separately.*—The initial symmetrical short-circuit current divides among the four generating stations A, B, C, and D as follows:

$$\text{From } A \text{ and } D \quad \frac{11{,}340 \times 4.63}{7.04} = 7{,}460 \text{ amp.}$$

$$\text{From } B \text{ and } C \quad = 3{,}880 \text{ amp.}$$

$$\text{From } A \quad \frac{7{,}460 \times 4.51}{7.76} = 4{,}340 \text{ amp.}$$

$$\text{From } D \quad = 3{,}120 \text{ amp.}$$

$$\text{From } B \quad \frac{3{,}880 \times 5.78}{9.70} = 2{,}310 \text{ amp.}$$

$$\text{From } C \quad = 1{,}570 \text{ amp.}$$

Obtaining the decrement factors from the curves (Fig. 20) plotted from data in tables I and II, the short-circuit currents supplied by each station at the end of 0.2 sec. are calculated in the table below.

Stations	Amperes at 22 kv.		Per cent equivalent reactance	Three-phase short circuit		Line-to-line short circuit	
	Rated	Calculated		Factor at 0.2 sec.	Amperes at 0.2 sec.	Factor at 0.2 sec.	Amperes at 0.2 sec.
A........	664	4,340	15.30	4.7	3,120	5.2	3,450
B........	985	2,310	42.6	2	1,970	2.2	2,170
C........	1,236	1,570	78.7	1.15	1,420	1.24	1,530
D........	3,530	3,120	113	0.82	2,900	0.82	2,900
Total....	.....	.....	......	....	9,410	....	10,050

The circuit breaker in the 22-kv. feeder will, hence, have to interrupt after 0.2 sec.

Three-phase short circuit..................... 9,400 amp.
Line-to-line short circuit..................... 10,000 amp.

2. *Application of Decrements to the Generating Stations Lumped.*—Total rated current capacity of the four stations A, B, C, and D combined

$$I_{\text{rated}} = 664 + 985 + 1{,}236 + 3{,}530 = 6{,}415 \text{ amp.}$$

$$\text{Equivalent reactance} = \frac{6{,}415 \times 100}{11{,}340} = 56.6 \text{ per cent}$$

From the decrement curves (Fig. 20) are obtained the following decrement factors at $t = 0.2$ sec.:

Three-phase short circuit.......................... 1.55
Line-to-line short circuit.......................... 1.69

The circuit breaker in the feeder will interrupt after 0.2 sec.:

Three-phase short circuit.................. $6{,}415 \times 1.55 = 9{,}940$ amp.
Line-to-line short circuit.................. $6{,}415 \times 1.69 = 10{,}830$ amp.

CHAPTER II

TRANSFORMER IMPEDANCE AND EQUIVALENT CIRCUITS

General Theory of Multicircuit Transformers. Consider a transformer having its coils connected in such a manner that there exist n independent circuits. In this n-circuit transformer, each circuit has a definite resistance, self-inductance, and mutual inductance with respect to every one of the other circuits. Employing the classical equations for coupled circuits, the instantaneous terminal voltage of each winding can readily be expressed in terms of these constants and the instantaneous currents in the various windings. The following equations result:

$$v_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + M_{12}\frac{di_2}{dt} + M_{13}\frac{di_3}{dt} + \cdots + M_{1n}\frac{di_n}{dt} \qquad (1)$$

$$v_2 = R_2 i_2 + L_2 \frac{di_2}{dt} + M_{21}\frac{di_1}{dt} + M_{23}\frac{di_3}{dt} + \cdots + M_{2n}\frac{di_n}{dt} \qquad (2)$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$v_n = R_n i_n + L_n \frac{di_n}{dt} + M_{n1}\frac{di_1}{dt} + M_{n2}\frac{di_2}{dt} + M_{n3}\frac{di_3}{dt} + \cdots \qquad (3)$$

In these equations, R and L with appropriate subscripts represent the resistance and self-inductance of the various circuits and M the mutual inductance between the two windings designated by the double subscript attached. It should be noted that the order of these subscripts is insignificant, *viz.*, $M_{1n} = M_{n1}$, etc.

If the n-circuit transformer considered is an air-core transformer, the self- and mutual inductances are strictly constant. If the transformer has an iron core, as is always the case in commercial practice, the self- and mutual inductances are variables, being functions of the saturation and, hence, of the instantaneous currents. The difficulty caused by the non-constancy of these fundamental parameters, which is inherent with iron-core transformers, may, in a great many problems, be removed by the following device:

The magnetic fluxes which give rise to the flux linkages corresponding to the self- and mutual inductances may be divided into two components, one which is confined to the iron core and another which wholly or partly exists in air. The former is by far the greater portion of the total flux, but the latter and smaller part determines almost entirely the operating characteristics of the transformer. It will now be assumed that the flux confined to the iron core depends only upon the *value* of the magnetomotive force producing it and is entirely unaffected by the *position* of this magnetomotive force with respect to the core. While this is not precisely true, the error should be very small indeed. In other words, the iron flux is assumed to be the same whether produced by a given number of ampere turns in circuit one, two, three or n.

Let the flux in the iron contribute the part M_c of the mutual nductances. Also, assume for simplicity that all windings have he same number of turns or that all constants involved are educed to the same base (referred to the same winding) in the vell-known manner by multiplying resistances and self-inducances by the squares of the proper ratios of turns and mutual nductances by the direct ratios of turns. Equations (1), (2), nd (3) may then be rewritten as follows:

$$_1 = R_1 i_1 + (L_1 - M_c)\frac{di_1}{dt} + (M_{12} - M_c)\frac{di_2}{dt} + (M_{13} - M_c)\frac{di_3}{dt} +$$

$$\cdot \cdot + (M_{1n} - M_c)\frac{di_n}{dt} + M_c\frac{d}{dt}(i_1 + i_2 + i_3 + \cdot \cdot \cdot + i_n) \quad (4)$$

$$_2 = R_2 i_2 + (L_2 - M_c)\frac{di_2}{dt} + (M_{21} - M_c)\frac{di_1}{dt} + (M_{23} - M_c)\frac{di_3}{dt} +$$

$$\cdot \cdot + (M_{2n} - M_c)\frac{di_n}{dt} + M_c\frac{d}{dt}(i_1 + i_2 + i_3 + \cdot \cdot \cdot + i_n) \quad (5)$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$_n = R_n i_n + (L_n - M_c)\frac{di_n}{dt} + (M_{n1} - M_c)\frac{di_1}{dt} + (M_{n2} - M_c)\frac{di_2}{dt} +$$

$$M_{n3} - M_c)\frac{di_3}{dt} + \cdot \cdot \cdot + M_c\frac{d}{dt}(i_1 + i_2 + i_3 + \cdot \cdot \cdot + i_n) \quad (6)$$

In these equations, the inductances corresponding to the ifferences $(L_1 - M_c)$, $(M_{12} - M_c)$, etc., are *sensibly constant uantities*, since they are due to fluxes which fully or partly exist air. The last term in each equation represents the voltage

induced in each winding by the flux exclusively confined to t
iron core.

Considering voltages and currents of a *single frequency onl*
equations (4) to (6) inclusive may be rewritten in vector for

$$
\begin{aligned}
V_1 = R_1 I_1 + j\omega(L_1 - M_c)I_1 + j\omega(M_{12} - M_c)I_2 \\
+ j\omega(M_{13} - M_c)I_3 + \cdots + j\omega(M_{1n} - M_c) \\
+ j\omega M_c(I_1 + I_2 + I_3 + \cdots + I_n) \quad (
\end{aligned}
$$

$$
\begin{aligned}
V_2 = R_2 I_2 + j\omega(L_2 - M_c)I_2 + j\omega(M_{21} - M_c)I_1 \\
+ j\omega(M_{23} - M_c)I_3 + \cdots + j\omega(M_{2n} - M_c) \\
+ j\omega M_c(I_1 + I_2 + I_3 + \cdots + I_n) \quad (
\end{aligned}
$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$
\begin{aligned}
V_n = R_n I_n + j\omega(L_n - M_c)I_n + j\omega(M_{n1} - M_c)I_1 \\
+ j\omega(M_{n2} - M_c)I_2 + j\omega(M_{n3} - M_c)I_3 + \cdots \\
+ j\omega M_c(I_1 + I_2 + I_3 + \cdots + I_n) \quad (
\end{aligned}
$$

Introducing, in general,

$$X_{nn} = \omega(L_n - M_c) \quad (1$$

$$X_{n1} = X_{1n} = \omega(M_{n1} - M_c) = \omega(M_{1n} - M_c) \quad (1$$

$$X_{n2} = X_{2n} = \omega(M_{n2} - M_c) = \omega(M_{2n} - M_c) \quad (1$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

and

$$E_c = j\omega M_c(I_1 + I_2 + I_3 + \cdots + I_n) \quad (1$$

the equations for the terminal voltages reduce to

$$V_1 = (R_1 + jX_{11})I_1 + jX_{12}I_2 + jX_{13}I_3 + \cdots + jX_{1n}I_n + E \quad (14$$

$$V_2 = (R_2 + jX_{22})I_2 + jX_{21}I_1 + jX_{23}I_3 + \cdots + jX_{2n}I_n + E \quad (15$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$V_n = (R_n + jX_{nn})I_n + jX_{n1}I_1 + jX_{n2}I_2 + jX_{n3}I_3 + \cdots + E \quad (16$$

Here, X_{11} is the *self leakage reactance* of circuit 1, and X_{12} is th
mutual leakage reactance between circuits 1 and 2. These react
ances are due to fluxes which wholly or partly exist in air. Henc
they are *very nearly* constant and independent of saturatior
The significance of the other reactances is at once apparent.

By subtracting each of the above equations from the precedin
one, E_c is eliminated and a new set obtained giving the *differenc*

between the terminal voltages or the effective impedance drop of pairs of windings, as follows:

$$V_1 - V_2 = [R_1 + j(X_{11} - X_{21})]I_1 - [R_2 + j(X_{22} - X_{12})]I_2 + j(X_{13} - X_{23})I_3 + \cdots + j(X_{1n} - X_{2n})I_n \quad (17)$$

$$V_2 - V_3 = [R_2 + j(X_{22} - X_{32})]\, I_2 - [R_3 + j(X_{33} - X_{23})]I_3 + j(X_{21} - X_{31})I_1 + \cdots + j(X_{2n} - X_{3n})I_n \quad (18)$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$V_n - V_1 = [R_n + j(X_{nn} - X_{1n})]I_n - [R_1 + j(X_{11} - X_{n1})]I_1 + j(X_{n2} - X_{12})I_2 + j(X_{n3} - X_{13})I_3 + \cdots \quad (19)$$

The difference between the self-reactance of a winding and the mutual reactance between this and one of the other windings is the *true leakage reactance* of the first winding with respect to the other. Thus, $(X_{11} - X_{21})$ is the true leakage reactance of winding 1 with respect to winding 2. Similarly, $(X_{22} - X_{12})$ is the true leakage reactance of winding 2 with respect to winding 1, and, in general, $(X_{nn} - X_{mn})$ is the true leakage reactance of winding n with respect to winding m. The relative aspect of the leakage reactances should be carefully noted. The leakage reactance of a winding is not a quantity which is dependent upon and characteristic of that winding alone; it must, of necessity, be defined with respect to some other winding. In an n-winding transformer, therefore, the true leakage reactance of one of the windings may, in general, have $n - 1$ values, namely, a distinct value for each of the other windings with respect to which the leakage reactance is determined. Of course, there is a possibility that two or more of these values may coincide, due, for instance, to symmetrical arrangement of the windings.

It is convenient to introduce symbols for the true leakage impedances and reactances as follows:

$$Z_{1(2)} = R_1 + jX_{1(2)} = R_1 + j(X_{11} - X_{21}) \quad (20)$$

$$Z_{2(1)} = R_2 + jX_{2(1)} = R_2 + j(X_{22} - X_{12}) \quad (21)$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$Z_{n(m)} = R_n + jX_{n(m)} = R_n + j(X_{nn} - X_{mn}) \quad (22)$$

$Z_{n(m)}$, for instance, represents the leakage impedance of winding n with respect to winding m. Hence, the first subscript refers to the winding itself, and the second one in parenthesis indicates the winding with respect to which the leakage reactance is considered.

Substituting these abbreviations, equations (17) to (1 inclusive become

$$V_1 - V_2 = Z_{1(2)}I_1 - Z_{2(1)}I_2 + j(X_{13} - X_{23})I_3 + \cdots + j(X_{1n} - X_{2n})I_n \quad (2$$

$$V_2 - V_3 = Z_{2(3)}I_2 - Z_{3(2)}I_3 + j(X_{21} - X_{31})I_1 + \cdots + j(X_{2n} - X_{3n})I_n \quad (2$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$V_n - V_1 = Z_{n(1)}I_n - Z_{1(n)}I_1 + j(X_{n2} - X_{12})I_2 + j(X_{n3} - X_{13})I_3 + \cdots \quad (2$$

The sum of the currents in the several windings is equal to tl exciting current. Hence, the following relation holds:

$$I_1 + I_2 + I_3 + \cdots + I_n = I_e \quad (2$$

In many practical problems, however, the exciting current ignored, since it is small compared with the load currents. (this basis, the sum of the currents is zero, *i.e.*,

$$I_1 + I_2 + I_3 + \cdots + I_n = 0 \quad (2$$

The voltage equations previously given (equations (14) to (1(or (23) to (25) inclusive) used in conjunction with one of tl current equations (equations (26) or (27)) suffice for the solutio of any single-phase or polyphase transformer problem. Of cours the constants involved, as well as a sufficient number of termin conditions, must be known.

In the following, the general theory will be specifically applie to two-circuit, three-circuit, and four-circuit transformers.

Two-circuit Transformers. *Application of the General Equa tions.*—In the case of a two-circuit transformer, the gener equations (14) to (16) inclusive reduce to

$$V_1 = (R_1 + jX_{11})I_1 + jX_{12}I_2 + E_e \quad (28$$

$$V_2 = (R_2 + jX_{22})I_2 + jX_{21}I_1 + E_e \quad (29$$

The difference between the terminal voltages or the effectiv impedance drop becomes

$$V_1 - V_2 = [R_1 + j(X_{11} - X_{21})]I_1 - [R_2 + j(X_{22} - X_{12})]I_2 = Z_{1(2)}I_1 - Z_{2(1)}I_2 \quad (30$$

Here, $Z_{1(2)}$ and $Z_{2(1)}$ are the true leakage impedances o windings 1 and 2, respectively, with respect to the other winding Since, in a two-winding transformer, however, there can neve be any doubt about the proper relative aspect of these imped ances, the subscripts in parentheses may be omitted, thus simpli fying the notation. With multicircuit transformers, on the othe

hand, the double subscripts must be retained. Equation (30) may, hence, be written

$$V_1 - V_2 = Z_1 I_1 - Z_2 I_2 \tag{31}$$

If excitation is taken into consideration, the sum of the primary and secondary currents equals the exciting current

$$I_1 + I_2 = I_e \tag{32}$$

Combining equations (31) and (32) gives

$$\begin{aligned} V_1 - V_2 &= (Z_1 + Z_2)I_1 - Z_2 I_e \\ &= -(Z_1 + Z_2)I_2 + Z_1 I_e \end{aligned} \tag{33}$$

In practical calculations, the exciting current is frequently neglected. This is permissible in very many problems, since the exciting current seldom exceeds 5 per cent of the full-load current and, hence, exerts but a small influence on the actual values of current, power, losses, and efficiency. Ignoring excitation, equation (33) reduces to

$$\begin{aligned} V_1 - V_2 &= (Z_1 + Z_2)I_1 = -(Z_1 + Z_2)I_2 \\ &= Z_{12} I_1 = -Z_{12} I_2 \end{aligned} \tag{34}$$

Here, Z_{12}, equal to the sum of the separate leakage impedances, represents the equivalent impedance of the transformer. This quantity is ordinarily determined by the standard short-circuit test and is the only constant required when the excitation is not taken into account. The composition of the equivalent impedance is at once apparent from the following equation:

$$Z_{12} = R_e + jX_e = R_1 + R_2 + j(X_1 + X_2) \tag{35}$$

Equivalent Network of Two-circuit Transformers.—Equations (31) and (32) indicate that a T-circuit, as shown in Fig. 21, is the logical equivalent network of a two-circuit transformer. The separate leakage impedances Z_1 and Z_2 make up the arms of the T, while the impedance of the pillar Z_c carrying the exciting current is given by

$$Z_c = \frac{V_1 - I_1 Z_1}{I_e} \tag{36}$$

In general, the separate leakage impedances of the two windings are not equal and the equivalent T-circuit representing the transformer will consequently be dissymmetrical. Very often, however, it is assumed that the equivalent impedance splits equally between the two windings. This assumption is fre-

quently necessary, since enough data for a correct determination of the separate impedances are seldom at hand. Fortunately, the symmetrical T-circuit thus obtained is sufficiently accurate for most practical problems, with the exception of such as specifically deal with the distribution of the harmonic components of the exciting currents between primary and secondary circuits.

Making use of equation (28), equation (36) may also be written

$$Z_c = \frac{E_c + jX_{12}I_e}{I_e} = \frac{E_c}{I_e} + jX_{12} \tag{37}$$

which again may be modified to

$$Z_c = \frac{j\omega M_c I_e}{I_e} + j\omega(M_{12} - M_c) = j\omega M_{12} \tag{38}$$

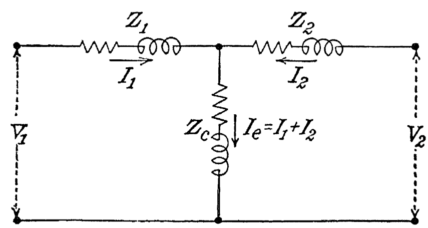


FIG. 21.—Equivalent network of a two-circuit transformer.

Equation (38) shows that, basing the constants of the equivalent circuit on the general theory, the pillar impedance Z_c contains reactance only. Furthermore, the value of this reactance corresponds to the mutual inductance between the two windings. It will be constant, therefore, only when the mutual inductance is a constant quantity or, what amounts to the same thing, where the magnetization curve is a straight line. Hence, the representation will be exact for air-core transformers. For iron-core transformers, the equivalent circuit is exact only at the particular value of saturation (voltage) at which the impedance Z_c is determined. At other values of saturation (voltage), the representation is, of necessity, more or less approximate, depending upon the shape of the magnetization curve and the position of the operating point on the latter.

Although it might be desirable from the standpoint of accuracy to use several values for Z_c when the equivalent circuit is used for the determination of performance at widely different voltages, this is usually not done. The reason is, of course, that the effect of the exciting current is fundamentally small. It is customary,

therefore, to determine the pillar impedance only at a saturation corresponding to normal voltage.

As already mentioned, equation (38) shows the impedance Z_c to contain nothing but reactance. This is due to the fact that the classical theory of coupled circuits fails to consider the effect of hysteresis and eddy currents in the iron. The equivalent circuit, therefore, derived directly from this theory, makes no provision for the proper consideration of the core loss. This loss may be accounted for by assigning also a resistance to the pillar impedance. This resistance is given such a value that the fictitious copper loss developed in it by the exciting current is equal to the core loss. Obviously, exact correspondence is also here obtained only at some particular value of voltage, since the relation between core loss and saturation is not exactly quadratic.

In practice, the pillar impedance, or excitation impedance (admittance), as it is commonly termed, is determined by an open-circuit test; *i.e.*, one winding is excited while the other is left open and, hence, carries no current. For this condition, equation (36) becomes

$$Z_c = \frac{V_1}{I_e} - Z_1 = (R_1 + j\omega L_1) - Z_1 \qquad (39)$$

The leakage impedance Z_1 is negligible compared to the mpedance $R_1 + j\omega L_1$ corresponding to the self-inductance. To ubstantiate this, consider a transformer having 10 per cent quivalent impedance and 5 per cent exciting current. The sum f the excitation impedance and the primary leakage impedance rom equation (39) corresponding to 5 per cent exciting current s 2,000 per cent. Assuming that the primary leakage impedance quals one-half the equivalent impedance, the actual value of the xcitation impedance becomes 1,995 per cent. Obviously, it is nmaterial whether this impedance is considered to be 1,995 or ,000 per cent, the difference between the two values being but ne-quarter of 1 per cent. Hence, the excitation impedance is ound by simply dividing the normal impressed voltage by the citing current, giving

$$Z_c = R_c + jX_c = \frac{V_1}{I_e} \qquad (40)$$

nd

$$Y_c = \frac{1}{R_c + jX_c} = \frac{I_e}{V_1} \qquad (41)$$

Since a T-circuit is always convertible into an equivalent Π-circuit, the transformer may evidently also be represented by the latter type of network. This is seldom done, however, since, in general, the T representation is more convenient.

When the consideration of excitation is omitted, the transformer circuit reduces to a single impedance. This impedance is equal to the equivalent impedance Z_{12} of the transformer, as indicated by equation (34).

There are but few problems involving transformers which actually *require* that the exciting current be taken into account. This is true when the analysis is concerned with the conditions in the power circuit alone. Where inductive interference is involved, the exciting currents, and particularly their higher harmonic components, may be just the currents that should be considered.

As examples of power problems where the effect of the excitation should be included may be mentioned unbalanced operation of the Y-Y-connected transformer bank without primary neutral and long-distance transmission where transformer banks are connected to long lines of considerable capacitance. The former case is rather unimportant, since very few Y-Y connections of this type exist. The latter, however, is exceedingly important. The lagging exciting current of the transformers tends to reduce the effect of the leading charging current of the line, and, if neglected, an entirely erroneous picture may be had of the conditions at the transformer terminals.

Three-circuit Transformers.[1] *Application of the General Equations.*—In the case of a three-circuit transformer, the general equations (14) to (16) inclusive reduce to

$$V_1 = (R_1 + jX_{11})I_1 + jX_{12}I_2 + jX_{13}I_3 + E_c \qquad (42)$$
$$V_2 = (R_2 + jX_{22})I_2 + jX_{21}I_1 + jX_{23}I_3 + E_c \qquad (43)$$
$$V_3 = (R_3 + jX_{33})I_3 + jX_{31}I_1 + jX_{32}I_2 + E_c \qquad (44)$$

By taking differences between pairs of the above equations, the effective impedance drops in circuits 1 and 2; circuits 2 and 3 and circuits 3 and 1, respectively, become

[1] Boyajian, A., "Theory of Three-circuit Transformers." *Trans. A.I.E.E.*, p. 508, 1924.

Peters, J. F., "Three-winding Transformers," *Elec. Jour.*, pp. 12 and 71, 1925.

Discussion by W. V. Lyon in *Trans. A.I.E.E.*, p. 813, 1925.

$$V_1 - V_2 = [R_1 + j(X_{11} - X_{21})]I_1 - [R_2 + j(X_{22} - X_{12})]I_2 + j(X_{13} - X_{23})I_3 \quad (45)$$

$$V_2 - V_3 = [R_2 + j(X_{22} - X_{32})]I_2 - [R_3 + j(X_{33} - X_{23})]I_3 + j(X_{12} - X_{13})I_1 \quad (46)$$

$$V_3 - V_1 = [R_3 + j(X_{33} - X_{13})]I_3 - [R_1 + j(X_{11} - X_{31})]I_1 + j(X_{23} - X_{12})I_2 \quad (47)$$

The sum of the currents in the three circuits equals the exciting current, *viz.*

$$I_1 + I_2 + I_3 = I_e \quad (48)$$

Introducing this relation in equations (45), (46), and (47), these may be written

$$V_1 - V_2 = [R_1 + j(X_{11} - X_{21} + X_{23} - X_{13})]I_1 - [R_2 + j(X_{22} - X_{12} + X_{13} - X_{23})]I_2 + j(X_{13} - X_{23})I_e \quad (49)$$

$$V_2 - V_3 = [R_2 + j(X_{22} - X_{32} + X_{13} - X_{12})]I_2 - [R_3 + j(X_{33} - X_{23} + X_{12} - X_{13})]I_3 + j(X_{12} - X_{13})I_e \quad (50)$$

$$V_3 - V_1 = [R_3 + j(X_{33} - X_{13} + X_{12} - X_{23})]I_3 - [R_1 + j(X_{11} - X_{31} + X_{23} - X_{12})]I_1 + j(X_{23} - X_{12})I_e \quad (51)$$

The symbols Z_1, Z_2, and Z_3 will now be introduced to represent the relatively complex impedances in the brackets above, as follows:

$$Z_1 = R_1 + j(X_{11} - X_{12} + X_{23} - X_{13}) \quad (52)$$

$$Z_2 = R_2 + j(X_{22} - X_{32} + X_{13} - X_{12}) \quad (53)$$

$$Z_3 = R_3 + j(X_{33} - X_{13} + X_{12} - X_{23}) \quad (54)$$

It is interesting to note the composition of the reactance part of these impedances. Consider Z_1, for instance. Here $X_{11} - X_{12}$ is the leakage reactance of winding 1 with respect to winding 2. X_{23} and X_{13} are the mutual reactances between windings 2 and 3 and between windings 1 and 3, respectively. Hence, the effective or composite reactance associated with circuit 1 is the leakage reactance of circuit 1 with respect to circuit 2 plus the differential effect of circuit 3 upon circuits 1 and 2. If the third circuit were symmetrically located with respect to the other two circuits, its mutual effect upon each would be the same. X_{23} would equal X_{13}, and the effective reactance or circuit 1 would be its leakage reactance with respect to circuit 2 alone. Or, if the third circuit did not carry any current, the mutual reactances X_{23} and X_{13} would not enter into the effective reactance of the first circuit at all. The problem then immediately reduces to a two-circuit transformer problem.

Another interesting feature is the possibility of negative effective reactance. Again referring to the impedance Z_1, it is conceivable that the circuits might be so arranged that the mutual reactances between circuits 1 and 2 and between circuits 1 and 3 are relatively large in comparison with the mutual reactance between circuits 2 and 3. If the former are sufficiently predominant in magnitude, the effective reactance of Z_1 may become negative and have the effect of a capacitive reactance.

The difference between the mutual reactances appearing in connection with the exciting current in equations (49) to (51) inclusive may be written as follows:

$$j(X_{13} - X_{23}) = Z_2 - Z_{2(1)} = Z_{1(2)} - Z_1 \tag{55}$$
$$j(X_{12} - X_{13}) = Z_3 - Z_{3(2)} = Z_{2(3)} - Z_2 \tag{56}$$
$$j(X_{23} - X_{12}) = Z_1 - Z_{1(3)} = Z_{3(1)} - Z_3 \tag{57}$$

Hence, the mutual reactance differences in question can always be determined as the difference between the effective "three-circuit" impedance of one of the windings and the leakage impedance of the same winding with respect to one of the others.

Equations (49), (50), and (51) may now be written in the following simplified form:

$$V_1 - V_2 = Z_1I_1 - Z_2I_2 + (Z_2 - Z_{2(1)})I_e \tag{58}$$
$$V_2 - V_3 = Z_2I_2 - Z_3I_3 + (Z_3 - Z_{3(2)})I_e \tag{59}$$
$$V_3 - V_1 = Z_3I_3 - Z_1I_1 + (Z_1 - Z_{1(3)})I_e \tag{60}$$

These equations properly applied will solve any three-circuit transformer problem where a sufficient number of terminal conditions is known. They provide for taking the exciting current into account, if this refinement is desired. Although the exciting current varies slightly with the load, it would usually be considered constant and be given the value which corresponds to normal saturation. It would, hence, be taken equal to the current flowing when rated voltage is impressed on one of the windings, the other two being open.

The exciting current, however, is very seldom included in the calculations. As a rule, it is neglected. When this is the case, the currents in the three circuits add to zero, and equations (58) to (60) inclusive reduce to

$$V_1 - V_2 = Z_1I_1 - Z_2I_2 \tag{61}$$
$$V_2 - V_3 = Z_2I_2 - Z_3I_3 \tag{62}$$
$$V_3 - V_1 = Z_3I_3 - Z_1I_1 \tag{63}$$

Determination of Impedances.—The effective impedances (equations (52), (53), and (54)) to be assigned to the three windings are readily determined by three short-circuit tests. They are, hence, obtained at practically zero saturation. The equivalent impedance of pairs of windings are measured exactly as for the two-circuit transformer. Since the exciting current for this condition is entirely negligible, the currents in the two windings under test are equal and opposite. Assuming that power is supplied to circuit 1 with circuit 2 short-circuited, then to circuit 2 with circuit 3 short-circuited, and, finally, to circuit 3 with circuit 1 short-circuited, equations (61), (62), and (63) give

$$V_1 = (Z_1 + Z_2)I_1 = Z_{12}I_1 \tag{64}$$
$$V_2 = (Z_2 + Z_3)I_2 = Z_{23}I_2 \tag{65}$$
$$V_3 = (Z_3 + Z_1)I_3 = Z_{31}I_3 \tag{66}$$

The equivalent impedances Z_{12}, Z_{23}, and Z_{31} are thus obtained. The effective impedances of the three circuits are then calculated from

$$Z_1 + Z_2 = Z_{12} \tag{67}$$
$$Z_2 + Z_3 = Z_{23} \tag{68}$$
$$Z_3 + Z_1 = Z_{31} \tag{69}$$

Simultaneous solution of these equations gives

$$Z_1 = \frac{Z_{12} + Z_{31} - Z_{23}}{2} \tag{70}$$

$$Z_2 = \frac{Z_{23} + Z_{12} - Z_{31}}{2} \tag{71}$$

$$Z_3 = \frac{Z_{31} + Z_{23} - Z_{12}}{2} \tag{72}$$

Equivalent Network of Three-circuit Transformers.—It is imprac-icable to represent the three-circuit transformer by an exact quivalent network when the exciting current is taken into ccount. Inspection of equations (58) to (60) inclusive will now the futility of attempting such representation.

Not so, however, when the effect of excitation is omitted. quations (61) to (63) inclusive show that, in this case, the three-rcuit transformer may be represented by an equivalent Y-nnected network, as indicated in Fig. 22. This representation simple and very convenient in many instances.

In three-phase connections, the individual transformers may so be represented by Y-connected circuits of the type shown in

Fig. 22. A complete three-phase arrangement is indicated in Fig. 23. It should be noted, however, that this representation is not practical except when conditions are perfectly balanced so that the three-phase problem reduces to a single-phase problem. When unbalance is involved, the solution must be based on the equations themselves rather than on an equivalent network.

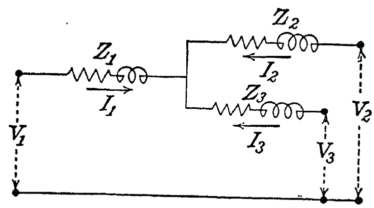


FIG. 22.—Equivalent network of a three-circuit transformer. Excitation neglected.

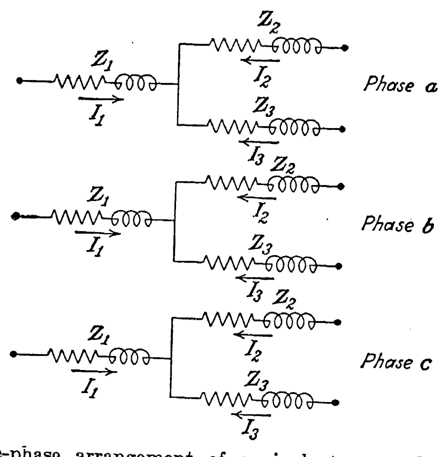


FIG. 23.—Three-phase arrangement of equivalent networks of three-circuit transformers. Excitation neglected.

Since a Y-connected circuit, in general, is convertible to a Δ-connected circuit, it is evident that the latter may also be used to represent a three-circuit transformer. As a rule, however, the equivalent Y-connected network is more convenient from the standpoint of calculation.

EXAMPLE 1

Statement of Problem

A 2,300-volt generating station supplies power to two short transmission lines through a bank of three 2,100 kv.-a. three-circuit transformers. The

transmission lines are both three-phase, and their voltage ratings are 110 and 22 kv., respectively. The transformers are connected Δ-Δ-Δ.

The nominal voltages of each single-phase transformer are as follows:

Winding 1...........	2,300 volts
Winding 2...........	22,000 volts
Winding 3...........	110,000 volts

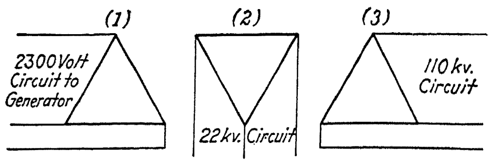


FIG. 24.—Three-circuit transformer layout for Example 1.

The equivalent short-circuit reactances are

6.10 per cent between the 2.3- and the 22-kv. windings
6.59 per cent between the 2.3- and the 110-kv. windings
6.15 per cent between the 22- and the 110-kv. windings

If the 110-kv. load at the transformer terminals is 3,150 kv.-a. at 85 per cent power factor (lagging), the 22-kv. load at the transformer terminals 3,150 kv.-a. at 90 per cent power factor (lagging), and the high-tension voltage strictly 110 kv., calculate

1. The voltage on the 22-kv. circuit.
2. The voltage of the generator.
3. The voltage regulation when the 110-kv. load is disconnected.

Solution

Ratios of Transformation:

$$\frac{\text{Winding 2}}{\text{Winding 1}} = \frac{22}{2.3} = 9.56$$

$$\frac{\text{Winding 3}}{\text{Winding 1}} = \frac{110}{2.3} = 47.8$$

$$\frac{\text{Winding 3}}{\text{Winding 2}} = \frac{110}{22} = 5$$

The circuit layout is shown in Fig. 24, and the equivalent Y-connected, single-phase network in Fig. 25. The reactances of the latter are (equations (70), (71), and (72))

$$X_1 = \frac{6.10 + 6.59 - 6.15}{2} = 3.27 \text{ per cent}$$

$$X_2 = \frac{6.15 + 6.10 - 6.59}{2} = 2.83 \text{ per cent}$$

$$X_3 = \frac{6.59 + 6.15 - 6.10}{2} = 3.32 \text{ per cent}$$

1. Consider the nominal voltages and the base currents calculated from the full rating at the nominal voltages to be the 100 per cent values of voltages and currents, respectively.

The voltage which would appear at the junction point of the equ circuit is given by

$$V_m = V_3 - jI_3X_3$$
$$I_3 = 50 \text{ per cent of base current}$$
$$\cos\phi_3 = 0.85 \quad \sin\phi_3 = 0.527$$

Using V_3 as standard phase,

$$V_m = 100 + 0.5(0.85 - j0.527)j3.32$$
$$= 100.88 + j1.41 = 100.9 \text{ per cent}$$

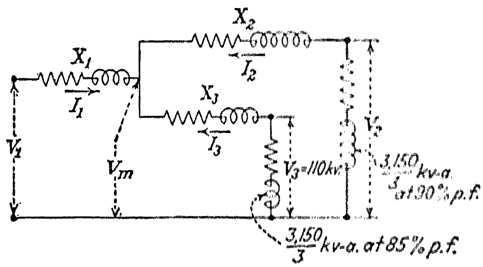


FIG. 25.—Equivalent network of one of the three-circuit transformers in with loads attached.

This voltage, however, is also given by

$$V_m = V_2 - jI_2X_2$$
$$I_2 = \frac{50 \times 100}{V_2} \text{ per cent of base current}$$
$$\cos\phi_2 = 0.90 \quad \sin\phi_2 = 0.436$$

Using V_2 as standard phase,

$$100.9\underline{/\alpha} = V_2 + \frac{50}{V_2}(0.9 - j0.436)j2.83 = V_2 + \frac{61.1}{V_2} + j\frac{127.2}{V_2}$$

Squaring gives

$$100.9^2 = V_2^2 + 122.2 + \frac{61.1^2}{V_2^2} + \frac{127.2^2}{V_2^2}$$

which again reduces to

$$V_2^4 - 10{,}050\,V_2^2 + 20{,}008 = 0$$

The solution of this equation is

$$V_2 = \sqrt{5{,}025 \pm \sqrt{25.23 \times 10^6 - 0.02 \times 10^6}}$$
$$= \sqrt{5{,}025 \pm 5{,}021} = 100.2 \text{ per cent}$$

The voltage on the 22-kv. circuit is, therefore,

$$22 \times 1.002 = 22.04 \text{ kv.}$$

2. As seen, the phase displacement between the voltage at the ju point V_m and the terminal voltage V_3 is very small, being less than 0. The displacement between V_m and V_2 is still smaller. It is suffic accurate, therefore, particularly since the resistances have been negl to consider the phase angle of the currents with respect to the junction voltage as being the same as with respect to the terminal voltages.

$$I_2 = \frac{50 \times 100}{100.2} = 49.9 \text{ per cent}$$
$$I_1 = -(I_2 + I_3)$$
$$I_1 = 49.9(0.9 - j0.436) + 50(0.85 - j0.527)$$
$$= 86.4 - j48.1$$

The generator voltage is given by

$$V_1 = V_m + jI_1X_1 \qquad (d)$$

Using V_3 as standard phase,

$$V_1 = 100.88 + j1.41 + (0.864 - j0.481)j3.27$$
$$= 102.45 + j4.23 = 102.5 \text{ per cent}$$

The voltage at generator is, hence,

$$2{,}300 \times 1.025 = 2{,}360 \text{ volts}$$

3. In calculating the regulation, it will be assumed that the size and power factor of the 22-kv. load remains the same in spite of the change in voltage.

With no load on the 110-kv. circuit the following equation holds:

$$V_1 = V_2 + I_1Z_{12} = V_2 - I_2Z_{12} \qquad (e)$$

Using V_2 as standard phase

$$102.5\underline{/\beta} = V_2 + \frac{50}{V_2}(0.9 - j0.436)j6.10 = V_2 + \frac{133}{V_2} + j\frac{274.3}{V_2}$$

Hence,

$$102.5^2 = V_2^2 + 266 + \frac{133^2}{V_2^2} + \frac{274.3^2}{V_2^2}$$

which reduces to

$$V_2^4 - 10{,}230\,V_2^2 + 93{,}100 = 0$$

The solution of this is

$$V_2 = \sqrt{5{,}115 \pm \sqrt{26.19 \times 10^6 - 0.09 \times 10^6}}$$
$$= \sqrt{5{,}115 \pm 5{,}108} = 101.1 \text{ per cent}$$

The regulation of the 22-kv. circuit is, therefore,

$$\frac{(101.1 - 100.2)100}{100.2} = 0.9 \text{ per cent}$$

The voltage of the 110-kv. windings is given by

$$V_3 = V_1 - I_1Z_1 \qquad (f)$$
$$I_1 = I_2 = \frac{50 \times 100}{101.1} = 49.4 \text{ per cent}$$

Again neglecting the small displacement between the voltages V_1 and V_2 as far as the phase of the current is concerned and considering V_1 as standard phase, equation (*f*) gives

$$V_3 = 102.5 - 0.494(0.9 - j0.436)j3.27$$
$$= 101.8 - j1.45 = 101.9 \text{ per cent}$$

The regulation of the 110-kv. circuit is, hence,

$$101.9 - 100 = 1.9 \text{ per cent}$$

Four-circuit Transformers. *Application of the General Equations.*—In the case of a four-circuit transformer, the general equations (14) to (16) reduce to[1]

$$V_1 = I_1(R_1 + jX_{11}) + jX_{12}I_2 + jX_{13}I_3 + jX_{14}I_4 + E_c \qquad (73)$$
$$V_2 = I_2(R_2 + jX_{22}) + jX_{12}I_1 + jX_{23}I_3 + jX_{24}I_4 + E_c \qquad (74)$$
$$V_3 = I_3(R_3 + jX_{33}) + jX_{13}I_1 + jX_{23}I_2 + jX_{34}I_4 + E_c \qquad (75)$$
$$V_4 = I_4(R_4 + jX_{44}) + jX_{14}I_1 + jX_{24}I_2 + jX_{34}I_3 + E_c \qquad (76)$$

Also, neglecting excitation,

$$I_1 + I_2 + I_3 + I_4 = 0 \qquad (77)$$

[1] See discussion by W. V. Lyon in *Trans. A.I.E.E.*, p. 816, 1925.

These five fundamental equations may be handled in a v of ways. Since there is some advantage in having th symmetrical form, the following method of elimination w used: First eliminate E_c by taking successive differences. eliminate I_4 from the first difference, I_1 from the second ence, and I_2 from the third difference. This gives

$$
\begin{aligned}
V_1 - V_2 = {} & I_1[R_1 + j(X_{11} - X_{12} - X_{14} + X_{24})] - \\
& I_2[R_2 + j(X_{22} - X_{12} + X_{14} - X_{24})] + \\
& \qquad j(X_{13} - X_{23} + X_{34} - X_{14})I_3 \\
V_2 - V_3 = {} & I_2[R_2 + j(X_{22} - X_{23} - X_{12} + X_{13})] - \\
& I_3[R_3 + j(X_{33} - X_{23} + X_{12} - X_{13})] + \\
& \qquad j(X_{13} - X_{34} + X_{24} - X_{12})I_4 \\
V_3 - V_4 = {} & I_3[R_3 + j(X_{33} - X_{34} - X_{23} + X_{24})] - \\
& I_4[R_4 + j(X_{44} - X_{34} + X_{23} - X_{24})] + \\
& \qquad j(X_{24} - X_{14} + X_{13} - X_{23})I_1
\end{aligned}
$$

These three equations together with equation (77) are suff to determine the currents, in any case. An examination of equations shows some interesting facts in regard to the ir ances. In the first equation, $R_1 + j(X_{11} - X_{12} - X_{14} + X_{24})$ impedance that would be assigned to the first winding if the second, and fourth windings were considered as a three-c transformer. This impedance will be represented by Z_{124}. impedance $R_2 + j(X_{22} - X_{12} + X_{14} - X_{24})$ is that which wou assigned to the second winding if the first, second, and f were considered as a three-circuit transformer. This w represented by Z_{214}. The first subscript shows to which wi the impedance is attached. The second and third subs indicate which of the other windings are grouped with the to form a three-circuit transformer. The order of the secon third subscripts is unimportant; that is, there is no diffe between Z_{124} and Z_{142}. It will also be noticed that the coeffi of I_3 in equation (78) is $Z_{213} - Z_{214}$, that the coefficient of equation (79) is $Z_{342} - Z_{312}$, and that the coefficient of equation (80) is $Z_{413} - Z_{423}$. Thus, the equations of vo differences may be written as follows:

$$
\begin{aligned}
V_1 - V_2 &= I_1 Z_{124} - I_2 Z_{214} + I_3(Z_{213} - Z_{214}) \\
V_2 - V_3 &= I_2 Z_{231} - I_3 Z_{321} + I_4(Z_{342} - Z_{312}) \\
V_3 - V_4 &= I_3 Z_{342} - I_4 Z_{432} + I_1(Z_{413} - Z_{423})
\end{aligned}
$$

There are some other interesting relations between these impedances. For example,

$$Z_{124} - Z_{123} = -Z_{214} + Z_{213} \tag{84}$$
$$Z_{134} - Z_{132} = -Z_{314} + Z_{312} \tag{85}$$
$$Z_{421} - Z_{423} = -Z_{241} + Z_{243} \tag{86}$$

Determination of Impedances.—Since the impedances involved in the solution of four-circuit transformer problems are the same as in the three-circuit case, they can be determined in a similar manner. Having obtained by test the equivalent impedances of pairs of windings, the desired effective impedances are calculated by

$$Z_{124} = \frac{Z_{12} + Z_{41} - Z_{24}}{2} \tag{87}$$

$$Z_{214} = \frac{Z_{24} + Z_{12} - Z_{41}}{2} \tag{88}$$

$$Z_{213} = \frac{Z_{23} + Z_{12} - Z_{31}}{2} \tag{89}$$

$$Z_{321} = \frac{Z_{31} + Z_{23} - Z_{12}}{2} \tag{90}$$

$$Z_{342} = \frac{Z_{34} + Z_{23} - Z_{24}}{2} \tag{91}$$

$$Z_{432} = \frac{Z_{24} + Z_{34} - Z_{23}}{2} \tag{92}$$

$$Z_{413} = \frac{Z_{34} + Z_{41} - Z_{31}}{2} \tag{93}$$

Very little, if anything, is gained by attempting to represent the four-circuit transformer by an equivalent network.

Determination of Separate Leakage Reactances.—The separate leakage reactances of transformer windings cannot, in general, be calculated with accuracy. The standard formulas found in textbooks on principles and design of transformers are all based on broad assumptions in regard to the distribution of the leakage flux and may easily give results which are in error to a considerable extent. Furthermore, it seems to be doubtful whether more rigorous and reliable formulas are capable of being developed.

For precise determination of these reactances, therefore, experimental methods must be resorted to. Theoretically, there are several tests for this purpose, some single-phase[1] and some

[1] Boyajian, A., "Resolution of Transformer Reactance into Primary and Secondary Reactances," *Trans. A.I.E.E.*, p. 805, 1925.

three-phase.[1] Some of these tests give the leakage reactance operating flux density (or any desired value of flux density, that matter), while some determine the leakage reactances a very low (practically zero) saturation. The latter correspo closely to the conditions under which the equivalent impedanc obtained by a short-circuit test. Since there undoubtedly *some* change in the leakage reactances when there is a mate change in saturation, these various tests may not give identica the same results.

When the saturation is decreased from that corresponding normal operation to approximately zero, the value of the equi lent reactance will increase, due to the reduction in reluctance the iron paths of the leakage fluxes. It is believed that increase in equivalent reactance will not exceed 10 per cent a usually will be less than this figure. The separate leaka reactances, therefore, will also increase but not necessarily in same proportion, since, as a rule, a large drop in saturation v be accompanied by a slight redistribution of leakage reacta between the windings. In a core-type transformer with cylind cal coils, for instance, a change in saturation affects the leaka reactance of the winding nearest to the core to a larger exte than the leakage reactance of the winding farther away from t core. This is exactly what might be expected, since the leaka flux of the former has a relatively longer path in iron than t leakage flux of the latter.

It should be noted that leakage-reactance changes of the orc of magnitude mentioned above manifest themselves only wh rather large changes occur in the flux densities. In particul some change in the leakage reactances may be expected when o value of the flux density corresponds to a condition where the ir is more or less saturated, while, at the other value, the effect saturation is absent; in other words, where the two operati points in question lie on *each* side of the bend of the magnetizati curve. It should also be noted, however, that even for qu considerable changes of flux density in the operating region, *i.* *above* the knee of the magnetization curve, *the leakage reactan remains sensibly constant.*[2]

[1] DAHL, O. G. C., "Separate Leakage Reactance of Transformer Wir ings," *Trans. A.I.E.E.*, p. 785, 1925.

[2] Substantiation of this statement will be found in the experimen described in the paper "Separate Leakage Reactance of Transformer Wir ings," by O. G. C. DAHL, *loc. cit.*

Evidently, it is desirable to ascertain the values of the leakage reactances at as nearly operating density as possible. Hence, from this standpoint, the tests which determine them at a high density are preferable. All of the tests, however, are not likely to give the same inherent precision. In calculating the leakage impedances from test data, *voltages and currents of a single frequency must be used.* When the waves are distorted, oscillograms must be taken and the components of the desired frequency singled out by analysis. In some of the tests, it is theoretically immaterial which one of the harmonics is used. Making use of the component which is the largest percentage of the composite wave, however, will give the maximum accuracy. In some tests, the largest component may be the fundamental; in others, the third harmonic. From the standpoint of precision of measurement, therefore, the best tests are either those which involve metering of pure waves of either fundamental or higher-harmonic frequency or those where any distortion likely to occur will be so small that the principal component of the wave is capable of exact determination by oscillogram analysis.

Single-phase Tests.—There are two single-phase tests which will give the actual values of the separate leakage impedances and one which will give their ratio. The latter, therefore, will suffice only when the sum of the leakage impedances or the equivalent impedance is known. The single-phase tests are applicable to both two-circuit and multicircuit transformers and may be used to give the leakage impedance of one winding with respect to any other winding. The determination of the separate leakage impedances is fundamentally a two-circuit problem, even in a multi-circuit transformer and, in outlining the tests below, two-circuit transformers will be assumed. If it is desired to determine all the possible separate leakage impedances involved in a multicircuit transformer, it merely means repetition of the same tests with appropriate changes of connections, so that each winding is considered with respect to every one of the others.

It is assumed that the transformers considered have unity ratio of transformation. If the ratio is different from unity, potential transformers must be used in the test involving measurement of voltage drop due to exciting current and in the parallel-conjunction test. A current transformer must be used in the series-opposition test. These auxiliary transformers should have the same ratio of transformation as the main transformer under test.

The auxiliary transformers may introduce errors due to incorre ratio and also due to the phase displacement between the primary and secondary voltages and currents.

a. Drop Due to Exciting Current. Excitation of One Windi Only.—In this test, one of the windings is excited, as show in Fig. 26, and the exciting current measured. One termin of the other winding is connected to the excited winding such a manner that the difference between the terminal vol

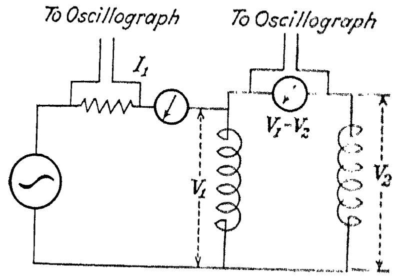


FIG. 26.—Single-phase determination of separate leakage impedances by meas uring voltage drop due to exciting current.

ages of the two windings can be measured as indicated with voltmeter drawing negligible current. Since the secondary wind ing carries no current, the voltage difference thus measured i directly equal to the primary leakage impedance drop due to th exciting current. The general equations are

$$V_1 = (R_1 + jX_{11})I_1 + jX_{12}I_2 + E_c \tag{94}$$
$$V_2 = (R_2 + jX_{22})I_2 + jX_{21}I_1 + E_c \tag{95}$$

For $I_2 = 0$, subtraction of these equations gives

$$V_1 - V_2 = (R_1 + j(X_{11} - X_{21}))I_1 = Z_1 I_1 \tag{96}$$

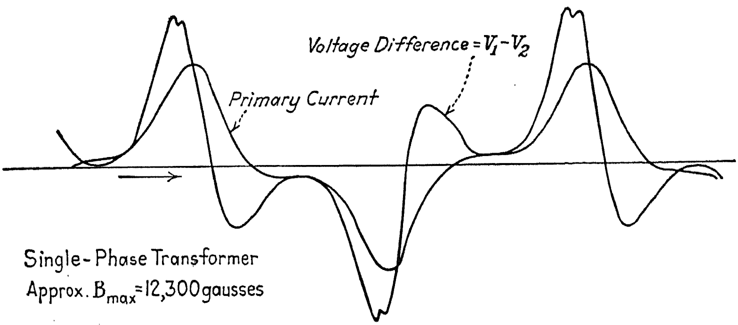
Hence,

$$Z_1 = \frac{V_1 - V_2}{I_1} \tag{97}$$

As seen, the leakage impedance of winding 1 with respect to winding 2 is immediately obtained. By repeating the test with winding 2 as primary and winding 1 as secondary, the leakage impedance of winding 2 with respect to winding 1 is also readily determined.

Both the measured current and the voltage drop will contain harmonics. Since the equations are applicable to quantities of a single frequency only, oscillograms must be taken and the desired harmonic components separated out by analysis. Figure 27

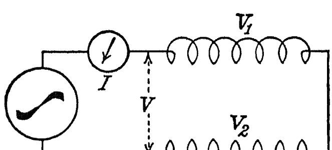
hows an oscillogram taken during a test of this type. The fun-amental component of the exciting current will always be the ırgest. Usually, also, the fundamental component of the oltage drop will be larger than any of the harmonics, except, erhaps, at very high saturations where the third-harmonic omponent may be the greatest. As a rule, therefore, maximum recision should be obtained by using the fundamental compo-



ɪɢ. 27.—Oscillogram from leakage-impedance test using the circuit connections shown in Fig. 26.

ents of voltage drop and current in calculating the leakage npedance.

Even though the fundamental components be used, however, ıe precision of this test leaves something to be desired. This is ue, of course, to the inherent difficulty of analyzing a complex ave with great accuracy. Only when the component which is anted is predominant can *exact* determination be expected.



Fɪɢ. 28.—Single-phase determination of separate leakage impedances by the ries-opposition test. Oscillographic measurements are not necessary in this se, as both voltages and currents are sinusoidal.

b. Series-opposition Test.—If the two windings are connected series opposition and an alternating-current voltage impressed, shown in Fig. 28, no flux will exclusively exist in the core. his is readily seen from the general equations which, with the indings in opposition, may be written

$$V_1 = (R_1 + jX_{11})I_1 - jX_{12}I_2 + j\omega M_c(I_1 - I_2) \qquad (98)$$
$$V_2 = (R_2 + jX_{22})I_2 - jX_{12}I_1 - j\omega M_c(I_1 - I_2) \qquad (99)$$

Since the two windings carry the same current, *i.e.*, since

$$I_1 = I_2 = I \tag{100}$$

the last term in equations (98) and (99) is zero, which means that there is no net magnetization of the core as a whole. These equations, therefore, reduce to

$$V_1 = (R_1 + jX_1)I = Z_1 I \tag{101}$$

$$V_2 = (R_2 + jX_2)I = Z_2 I \tag{102}$$

which show that the terminal voltage of each winding is directly equal to its separate leakage-impedance drop. The total voltage impressed on the two windings in series evidently equals the equivalent leakage-impedance drop. Thus,

$$V = V_1 + V_2 = (Z_1 + Z_2)I = Z_{12}I \tag{103}$$

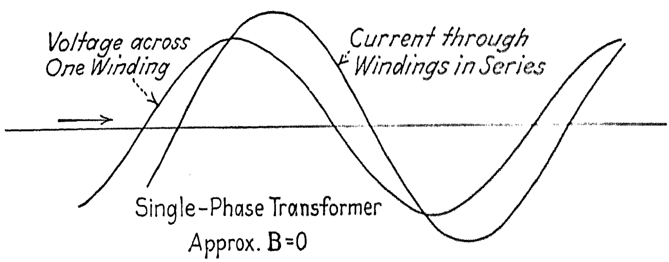


FIG. 29.—Oscillogram from leakage-impedance test by the series-opposition method. Note the sinusoidal shape of the voltage and current waves. Circuit connections shown in Fig. 28.

When the applied voltage is sinusoidal, the current flowing, as well as the terminal voltages of the two windings, will also be sinusoidal, since the flux in the core is suppressed. Exact measurements of the two voltages and the current involved are readily obtained, and oscillograms can be omitted. From the standpoint of precision, therefore, this test is very satisfactory. It gives, however, the leakage impedances at *zero saturation*, and the values obtained may be slightly larger than those corresponding to normal density.

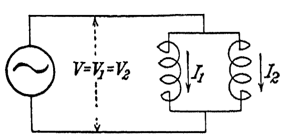


FIG. 30.—Single-phase determination of the ratio of separate leakage impedances by the parallel-conjunction test.

Figure 29 shows an oscillogram taken during a series opposition test of a small laboratory transformer. It will be noted that both voltage and current waves are pure sinusoids.

c. Parallel-conjunction Test.—With the windings excited in parallel conjunction as indicated in Fig. 30 the current will divide

etween the two windings in the inverse ratio of their leakage npedances.[1] The equations for this case are

$$V_1 = (R_1 + jX_{11})I_1 + jX_{12}I_2 + E_c \tag{104}$$
$$V_2 = (R_2 + jX_{22})I_2 + jX_{21}I_1 + E_c \tag{105}$$

hich, by subtraction, gives

$$0 = (R_1 + jX_1)I_1 - (R_2 + jX_2)I_2 = Z_1I_1 - Z_2I_2 \tag{106}$$

Ience,

$$\frac{I_1}{I_2} = \frac{Z_2}{Z_1} \tag{107}$$

This test may be performed at any desired value of saturation y varying the impressed voltage. Equation (107) is, in general, pplicable to a single frequency only, and, since the currents in iis test will always be more or less distorted, oscillographic ecords are necessary. The oscillograms Figs. 31 and 32, which ere taken when tests of this type were applied to a small laboraory transformer, serve to illustrate this point. As seen, promient harmonics are present even when the saturation is omparatively low (see Fig. 31).

[1] It might be thought that, with the windings in *parallel opposition*, the irrent would divide inversely as the leakage impedances and also that, ith this connection, the impressed voltage would equal each of the leakage ipedance drops. This is erroneous, however, as the following analysis will ow. The equations for this case are

$$V_1 = (R_1 + jX_{11})I_1 - jX_{12}I_2 + j\omega M_c(I_1 - I_2) \tag{a}$$
$$V_2 = (R_2 + jX_{22})I_2 - jX_{21}I_1 + j\omega M_c(I_1 - I_2) \tag{b}$$

Addition of these equations gives

$$V_1 + V_2 = 2V = Z_1I_1 + Z_2I_2 \tag{c}$$

$$V = \frac{Z_1I_1}{2} + \frac{Z_2I_2}{2} \tag{d}$$

Hence, the impressed voltage is equal to one-half the sum of the separate ikage-impedance drops of the two windings.

By equating equations (*a*) and (*b*), the current division is given by

$$\frac{I_1}{I_2} = \frac{R_2 + j(X_{22} + X_{12}) + 2j\omega M_c}{R_1 + j(X_{11} + X_{21}) + 2j\omega M_c} \tag{e}$$

iich is entirely different from the inverse leakage-impedance ratio.

Only when the two windings are identical in every respect and perfectly mmetrically arranged with respect to the core will the currents divide versely as the leakage impedances. The ratio of the currents in such a se would evidently be unity, and windings would have the same impedce. The last term in equations (*a*) and (*b*) would be zero (*i.e.*, the core as whole completely demagnetized), and the impressed voltage equal to the pedance drop of either winding.

Only when the leakage impedances contain no resistance (or, in practice, when the resistance is entirely negligible as compared to the reactance) can oscillograms be omitted. In this particula case, the ratio of the effective values of the distorted current wil

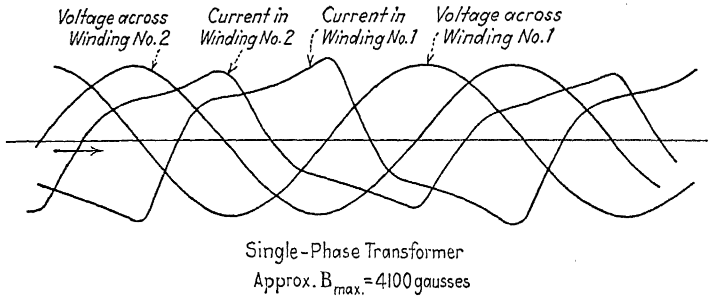


FIG. 31.—Oscillogram from leakage-impedance test by the parallel-conjunc tion method. Circuit connections shown in Fig. 30. A two-element oscillo graph was used, necessitating a separate exposure for each winding. Th apparent phase displacements, therefore, between the recorded quantities fo winding 1 and winding 2 have no real significance. The voltages should be ver nearly in phase. The voltage across winding 1 is reversed with respect to th current and represents a voltage rise instead of a drop.

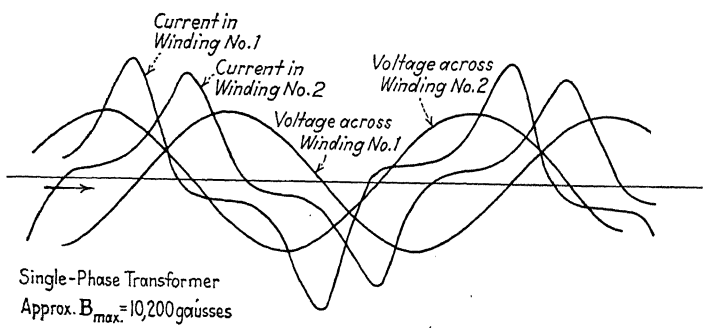


FIG. 32.—Oscillogram from leakage-impedance test by the parallel-conjunc tion method. Circuit connections shown in Fig. 30. A two-element oscillo graph was used, necessitating a separate exposure for each winding. Th apparent phase displacements, therefore, between the recorded quantities fo winding 1 and winding 2 have no real significance. The voltages should b very nearly in phase. The voltage across winding 1 is reversed with respec to the current and represents a voltage rise instead of a drop. Note that th distortion is larger than shown in Fig. 31 on account of increase in saturation.

equal the inverse ratio of the leakage reactances. Assumin fundamental and third-harmonic components only, the ratios ar as follows:

$$\frac{I_1'}{I_2'} = \frac{X_2'}{X_1'} = \frac{3X_2'}{3X_1'} = \frac{X_2'''}{X_1'''} = \frac{I_1'''}{I_2'''} \qquad (108$$

Hence, the ratio of the fundamental and the third-harmonic currents in the two windings is the same. This would be so also for the fifth harmonics, seventh harmonics, etc. The ratio of the effective currents may be written

$$\frac{I_1}{I_2} = \sqrt{\frac{(I_1')^2 + (I_1''')^2}{(I_2')^2 + (I_2''')^2}} \tag{109}$$

Substituting for I_2' and I_2''' by means of equation (108), equation (109) reduces to

$$\frac{I_1}{I_2} = \sqrt{\frac{(I_1')^2 + (I_1''')^2}{(I_1')^2\left(\frac{X_1'}{X_2'}\right)^2 + (I_1''')^2\left(\frac{X_1'}{X_2'}\right)^2}} = \frac{X_2'}{X_1'} \tag{110}$$

which indicates that the effective currents divide inversely as the leakage reactances.

The parallel-conjunction test gives the ratio only of the leakage impedances and not their actual values. When the equivalent impedance is known from other tests, however, it can be apportioned between the two windings, and the separate leakage impedances determined in this manner.

Three-phase Tests.—The three-phase method by which the separate leakage reactance of the windings of a transformer may be determined makes use of the third-harmonic component which inherently exists in the magnetizing current of a transformer when a sinusoidal voltage is impressed. The method is applicable only when a three-phase bank of three identical transformers is available.

The principle of the method is as follows: If sinusoidal voltages are impressed on a Y-Δ-connected bank of transformers, the third-harmonic component of the magnetizing current will be confined to the delta, where it appears as a circulating current. If the transformers are perfectly balanced and there is no external load on the secondary, no current other than the third harmonic and its multiples can exist in the delta. Usually, the ninth and fifteenth harmonics, etc., are negligible and need not be considered. The third-harmonic electromotive force induced per phase of the delta is just balanced by the triple-frequency impedance drop due to the circulatory third-harmonic current. The problem is then to measure with precision the proper third-harmonic electromotive force and current, which, by simple division, will give the desired triple-frequency leakage impedance.

The three-phase third-harmonic tests determine the leak: reactances at normal (or any desired) saturation. They sho: give good accuracy of measurement. Of course, due to unb ance and other causes, it may be impracticable to obtain entir pure waves; but, in any event, the quantities of triple frequer which it is desired to measure will be entirely predominant, a) hence, correct determination is highly facilitated even thou oscillogram analysis may be necessary. Furthermore, inst: ment transformers, if used, are not likely to affect the resu seriously, since their secondaries are connected directly indicating meters. All doubt in regard to the calibration of t instrument transformers is thus eliminated.

a. Two-winding Transformers.—The bank is connected Y· and balanced sinusoidal voltages impressed. The third-harmo) current in the delta and the third-harmonic electromotive fo) per phase on the primary side are recorded. The latter is m(

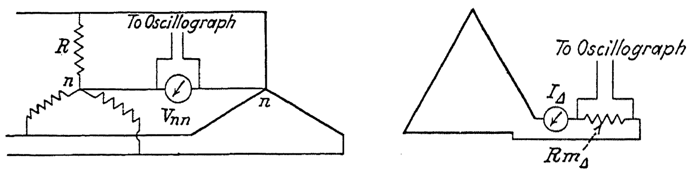


FIG. 33.—Diagram of connections for three-phase leakage-impedance test two-winding transformers.

practically obtained by connecting a Y-connected resistor ba) between the lines and measuring the voltage between the resist and transformer neutrals. The resistance of the voltmeter a) the bank of resistors should be sufficiently high so that t] primary third-harmonic current is negligible compared to t] current circulating in the delta.

If the generator is Y-connected and its phase voltage is fr from a third harmonic (and multiples), the bank of resistors m: be omitted and the voltage measured between generator a) transformer neutrals. No commercial Y-connected generatc however, is entirely without a third-harmonic component its voltage to neutral, so this method is scarcely of practic interest.

As a rule, it will be necessary to take oscillographic records a) separate out the third harmonics by analysis. While the curre)

in the delta is sensibly third harmonic, a fundamental and also other harmonics are unavoidable between the two neutrals, if even the slightest unbalance in the impressed voltages, the resistors, or the transformers themselves is present.

The diagram of connections and the third-harmonic vector diagram are given in Figs. 33 and 34, respectively. One to one ratio

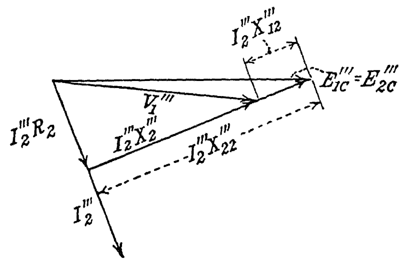


FIG. 34.—Vector diagram of third-harmonic quantities involved in the "two-winding method."

of transformation is assumed. If the transformers have another ratio, the quantities in the various equations given below should all be referred to the same side. Figure 35 shows an oscillogram taken during a test of a bank of experimental transformers.

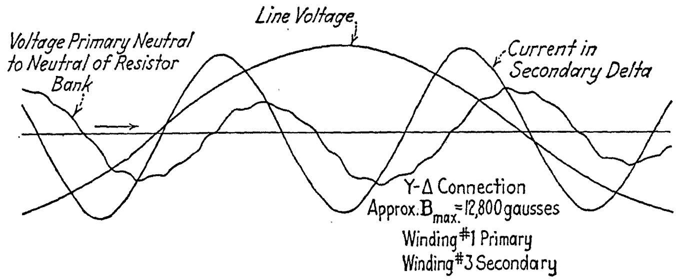


FIG. 35.—Oscillogram from three-phase leakage-impedance test on two-winding transformers. Circuit connections shown in Fig. 33.

If R is the resistance of the resistors per phase, r the resistance of the voltmeter, and V_{nn}''' the third-harmonic voltage between the neutrals, then

$$V_1''' = V_{nn}'''\left(1 + \frac{R}{3r}\right) \tag{111}$$

Since the third-harmonic current in the primary is zero (or, at least, negligibly small), this voltage is the sum of two compo-